Economic Scenario Generator  
Version 7 Release Notes  

These release notes describe the changes to the Academy’s Economic Scenario Generator that are included in version 7. This release includes updated versions of both the interest rate generator that has been used for C3 Phase 1 and the fund return generator used to produce the pre-defined scenario sets used for C3 phase 2. The fundamental structure of the stochastic processes used in both generators remains unchanged, but some details of implementation have been updated. The updates are listed below, with additional details in the attachments.

Updates to the interest rate generator
1. The floor on generated interest rates has been changed to 0.01%. Previously the floor was formulaic. The change was motivated by the fact that actual interest rates have fallen below the previous floor.
2. The method used to interpolate points on the yield curve has been changed. The new method is based on the Nelson-Siegel formula and is documented in an attachment.
3. The mean reversion point for the 20-year rate is now set using a dynamic formula specified by the NAIC. Details of the mean reversion point determination are documented in an attachment.

Updates to the fund return generator
1. The returns on fixed income funds are now based on the path of interest rates in the same numbered scenario. Since the path of interest rates depends on the starting yield curve, this means that the fixed income fund scenarios now depend on the starting yield curve.
2. The random number generator is different from the one used to generate the pre-defined scenarios that were originally released. The random number generator in use now is the same as that which has always been used by the interest rate generator.

Updates to the Excel implementation of the generators
1. Ease of use has been enhanced through a revised layout of the workbook. A User’s Guide is attached to explain how to use the workbook to generate scenarios.
2. A scenario-picker tool has been incorporated into the workbook, and is run automatically every time a complete set of 10,000 scenarios is generated.
3. An additional, optional file format for output has been enabled. Scenarios can be generated in EconSML, a format developed as a proposed standard by the Technology Section of the Society of Actuaries. More information on this format is available at http://www.soa.org/professional-interests/technology/tech-scenario-file-format.aspx.


Attachment 1

Yield Curve Interpolation Method

The Academy’s interest rate generator is based on a stochastic process that defines interest rates for 20-year maturities and 1-year maturities. An additional process must be defined to derive the other eight points on the yield curve that are included in the generator output. Version 7 of the generator uses a process based on a Nelson-Siegel formula. The change to this process was motivated by anomalous results that were sometimes produced by previous versions of the generator when the low interest rate environment of 2010 was used for starting conditions.

The Nelson-Siegel formula has four parameters:

\( b_0 \) = the ultimate long term rate
\( b_1 \) = the excess of the instantaneous short rate over the long term rate (normally negative)
\( b_2 \) = a parameter governing the size of hump in the yield curve
\( k \) = a parameter governing the curvature and location of hump in the yield curve

The interest rate for \( t \) years to maturity is calculated as follows. Of course \( t \) can be fractional when calculating yields for a number of months that is not a whole number of years.

\[
 r(t) = b_0 + b_1 \left( \frac{1 - e^{-kt}}{kt} \right) + \frac{b_2}{k} \left( \frac{1 - e^{-kt}}{kt} \right) - e^{-kt} \]

The Academy’s scenario generator generates yields for coupon paying bonds priced at par, or par curve, while this interpolation formula was originally developed to define the zero coupon rate curve, or zero curve. Since the par curve and the zero curve have very similar shape, the use of this formula as a basis of interpolation is felt to be appropriate.

It is a simple matter to fit the parameters of this formula to a yield curve that is defined by two points. When generating scenarios using the Academy’s generator, the generated future curves require interpolation between two points (1yr and 20yr).

The following procedure is used to interpolate yield curves in the Academy’s generator.

1. Fit the Nelson-Siegel parameters to the yield curve. To do this, set \( k = 0.4 \) and set \( b_2 = 0 \). Then solve two linear equations in two unknowns (\( b_0 \) and \( b_1 \)). The two equations are defined by the 1-year and 20-year rates from the generator’s stochastic process using the formula given above. Note that the coefficients of \( b_0 \) and \( b_1 \) are defined if \( k \) and \( b_2 \) are given and \( t \) (the years to maturity) is given.

2. Adjust for differences between the fitted starting curve and the actual starting curve. The Nelson-Siegel fitted curve will exactly reproduce only two of the 10 points on the starting yield curve. To provide a better transition from the starting curve to projected curves, the following procedure is used to adjust the fitted curves.
   a. For the starting curve, determine the difference between the fitted curve and the actual starting curve at each of the 10 points on the curve.
   b. Each month after the starting date, these differences are reduced by 1/12\(^{th}\) of their starting values, and the reduced differences are added to the fitted N-S curves. After 12 months, no adjustment is made to the fitted N-S curves.

This procedure is predicated on the idea that a “normal” yield curve shape is defined by the Nelson-Siegel formula with \( b_2 = 0 \) and \( k = 0.4 \). Setting \( b_2 \) to zero removes any hump, and setting \( k = 0.4 \) provides the normal nonlinear shape of the curve between short and long rates.
This procedure can under rare circumstances produce negative interest rates. To prevent that, all generated interest rates, at every point on the yield curve, are floored at 0.01% after the above procedure is carried out.
Attachment 2
The dynamic mean reversion point

The mean reversion point for the 20-year Treasury bond rate is no longer a fixed value. The mean reversion point is dynamic, based on historical interest rates as they emerge. The formula for the dynamic mean reversion point has been defined by the NAIC as follows:

\[
\text{Mean reversion point} = 0.20 \times \text{median of the last 600 months} + 0.30 \times \text{average of the last 120 months} + 0.50 \times \text{average of the last 36 months}
\]

The result is then rounded to the nearest 0.25%.

The mean reversion point for use in the generator changes once per year, in January, and is based on historical rates through the end of the prior year. While the mean reversion point is dynamic depending on the date from which a scenario starts, it remains constant (rather than dynamic) across all time periods after the scenario start date, for purposes of generating the scenario.

The historical 20-year Treasury bond rate for each month is the rate reported for the last business day of the month. Historical rates can be obtained from the US Treasury at http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield_historical_main.shtml

Calculated values of the mean reversion point appear on the “MRP” tab of the scenario generator workbook. The user will need to update values on the “Historical Curves” tab before the workbook can update the mean reversion point for scenario start dates after the workbook was created.

Many parts of the workbook are “protected,” meaning that the user cannot change them. This protection can easily be removed if the user wishes to do so, but it is understood that any changes made after removing such protection are at the user’s own risk. Protection can be removed (or reinstated) for any worksheet tab by clicking the “Unprotect Sheet” button on the Review ribbon in Excel 2007. Other versions of Excel also provide menu options to remove or reinstate protection. No password is required to remove protection in this workbook.
Attachment 3
User Guide to the scenario generator workbook

Overview
The scenario generator workbook has been created to enable the user to create collections of stochastic scenarios that are saved in computer files. The scenarios can be used in fulfilling regulatory requirements for valuation and testing reserve adequacy, as well as for general risk management work. The generator allows the user to create updated sets of scenarios that start from economic conditions as they change for each valuation date.

The scenarios produced by the generator include the future path of interest rates for US treasury securities as well as the future path of total returns on several kinds of investment portfolios, including both equity portfolios and fixed-income portfolios. The scenarios do not include interest rates on corporate securities or other types of fixed-income securities, nor do they include expected default rates or inflation rates. Where such additional items are required for use in modeling, the user must obtain them through other means.

The generator is set up to produce a full set of 10,000 scenarios, but provides a scenario picking tool that uses stratified sampling to select a smaller number of scenarios from the full set. The scenario picking tool is set up to provide samples of various sizes, including 1000 scenarios, 500 scenarios, and 200 scenarios. More information is included below in the section on the “Scenario Subsets” tab.

The generator can also produce the set of 16 scenarios used in the stochastic exclusion test. Further discussion of these scenarios appears in the section on the “Test Shocks” tab.

The generator workbook requires a small amount of user maintenance because the mean reversion point parameter of the generator is dynamic and depends on recent historical interest rates. See the documentation of the “Historical Rates” tab for instructions concerning entry of historical interest rates. If the user does not carry out this maintenance, the mean reversion point parameter will become outdated and potentially incorrect.

Using the “Scenario Generator” tab to generate scenarios
To generate sets of scenarios, the user must click the large “Generate Scenarios” button that appears near the top center of this tab. However, before doing so the user should specify all the information used by the generator when creating the scenarios. This means setting all of the following inputs that appear on the tab:

1. Set the starting date for each scenario, in YYYYMM form.
2. Set the US treasury yield curve on the starting date. The user must specify all 10 points shown. This can be done automatically by pressing the button labeled “Get starting curve from historical curves.” That button retrieves the starting yield curve from the “Historical curves” tab of the workbook, but only if the yield curve for the scenario starting date is available there. If the button is pressed but the desired yield curve is not available, a dialog explaining that fact will appear.
3. Set the mean reversion point. This is done automatically, using the NAIC definition of the dynamic mean reversion point. However, if you want to change the mean reversion point to a different value, you can do so as described under the “Parameters” tab.
4. Set the number of years to be included in each scenario. The generator allows use of time periods up to 150 years.
5. Set the number of digits to retain after the decimal. The default value is 5, which means interest rates and fund returns are saved to an accuracy of 0.001%, or one tenth of a basis point.
6. Select the folder where the generated files will be stored. One must click the button to select the folder using a standard Windows dialog box.
7. Specify an optional filename suffix. The generator produces several files for each collection of scenarios. This suffix will be added to the end of all filenames, just before the dot and the filename extension that indicates file type. The filename suffix can be left blank, but you may wish to use this as a means of distinguishing one set of scenario files from another set.
8. Specify which output files should be produced. This is done by checking one or more of the check boxes near the lower left corner of the worksheet page. Details concerning the files that are produced in response to each check box appear below, after this list of inputs.
9. Specify the time step. Note that the generator always works on a monthly time step. However, scenarios can be stored in a manner that includes every monthly value or just values at the end of each quarter, half-year, or full year.
10. Specify whether the output yield curves from the interest rate generator should be bond-equivalent yields or spot rates. The default is bond-equivalent yields, but some software may prefer spot rates instead.
11. Specify the number of scenarios to be generated by selecting one of the radio buttons on the top right of the tab.

After all of these inputs have been specified, the user can press the “Generate Scenarios” button to create files. Since scenario generation can take some time, the status bar at the bottom of the Excel window provides a continuously updated report on progress.

Generating scenarios will result in one or more files being created in the folder specified in step 6 above. The files that are created depend on the check boxes selected in step 8. The list of files created for each check box is shown below. The filenames shown here do not include the optional user-specified filename suffix.

<table>
<thead>
<tr>
<th>Checkbox</th>
<th>Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separate *.csv file for each term to maturity</td>
<td>UST_3m.csv UST_6m.csv UST_1y.csv UST_2y.csv UST_3y.csv UST_5y.csv UST_10y.csv UST_20y.csv UST_30y.csv AGGR.csv INT.csv US.csv SMALL.csv MONEY.csv INTGOV.csv LTCORP.csv FIXED.csv BALANCED.csv</td>
</tr>
<tr>
<td>Single file</td>
<td>UST.csv (this file contains interest rates of all maturities) AGGR.csv INT.csv US.csv SMALL.csv MONEY.csv INTGOV.csv LTCORP.csv FIXED.csv BALANCED.csv</td>
</tr>
</tbody>
</table>
EconSML file  AAA scenarios.xml  AAA scenarios.offsets.xml

Random numbers  FundNormalDraws.csv

To get random numbers used by the interest rate generator, you must check both the “Random numbers” box and either the “Single file” or “Separate .csv file” boxes.

With “Separate .csv file” checked  UST_Z1.csv  UST_Z2.csv  UST_Z3.csv

With “Single file” checked  InterestNormalDraws.csv

The format of the *.csv files is the same as has been used for earlier versions of Academy generators. The format the EconSML file (*.xml) is described at http://www.soa.org/professional-interests/technology/tech-scenario-file-format.aspx. Sample software for using files in that format is available at that site.

The “Single Scenario” tab
This tab is included purely for instructive purposes and is not a part of the generator itself. The tab is instructive because it includes all the calculations used to create a single interest rate scenario, exposed in Excel workbook form. The user can position the cursor over any intermediate result and see the formula that is used to calculate it.

The user can view the calculation of any specific scenario by changing the scenario number in cell F16. When this is done, the random number generator is called to fill columns D, E, and F with uncorrelated random numbers from a Gaussian (a.k.a. Normal) distribution with a mean of zero and variance of 1. All other calculations involved in generating the full yield curve for every month in the scenario are shown in the workbook. In addition, the calculation of the scenario’s “significance measure” is shown at the far right. The significance measure is used for stratified sampling by the scenario picker tool.

The “Scenario Subsets” tab and the scenario picker
Columns B – E on this tab contain lists of the scenario numbers (out of the complete set of 10,000) that can be used in four different-sized subsets, each of which represents a stratified sample. Since each scenario number corresponds to a unique random number seed, the selected scenario numbers can be used to generate the selected scenarios. These are the scenario numbers that are used when the user selects a subset of scenarios on the “Scenario Generator” tab.

Each time the generator is run to produce a full set of 10,000 scenarios, the scenario picking tool automatically ranks the scenarios and selects the scenario numbers for each subset (i.e., each size of stratified sample). The selected scenario numbers are saved in a file named ScenarioSubsets_YYYYMM_(suffix).txt where “YYYYMM” corresponds to the scenario start date and “(suffix)” corresponds to the optional filename suffix specified by the user in cell F23 of the Scenario Generator tab. This file is stored in the same folder as the scenario files themselves.

The scenario subsets can be retrieved from such a file by clicking the button that appears at the top of the page, labeled “Load subsets from a saved file.” Doing so will bring up a dialog box allowing
the user to select the file from which to load the subsets. If a file is selected, the lists of scenario numbers will be updated and used when generating subsets of the complete set of 10,000 scenarios.

It should be noted that the scenario picking tool included in this workbook is illustrative only. The user is free to use any other scenario reduction method to select scenario subsets. If, for this reason, the user needs to enter a custom list of scenario numbers, the user will need to remove the protection from this tab to allow user input. Various versions of Excel provide different menu options to “unprotect” a worksheet. In Excel 2007, this is available in the Review tab of the ribbon. No password is required to unprotect this sheet.

The “Parameters” tab
This tab shows all of the parameters used by the generators for both interest rates and fund returns. Parameters for the interest rate generator appear on the left and those for the fund return generators appear on the right.

This entire tab is protected to prevent the user from accidentally changing parameter values.

The mean reversion point parameter of the interest rate generator (cell C4) gets special treatment because it must be dynamic rather than fixed. The default value of this parameter is updated each time the scenario start date is changed on the “Scenario Generator” tab. The generator will use the default value unless the user checks the box at the top of column F. If that box is checked, the default value is not used, and the user-specified value in cell C4 is used. When the box is unchecked, the value in use reverts to the default.

The “MRP” tab
This tab is used to calculate the dynamic mean reversion point for the interest rate generator, based on the historical interest rates on the “Historical Rates” tab. The dynamic mean reversion point, as defined by the NAIC, appears in column J.

The user should not need to change anything on this tab.

The “Historical Curves” tab
This tab contains the historical yield curves for US treasuries that are used when updating the mean reversion point of the interest rate generator. The user is expected to maintain this tab by entering the yield curve for each month as time passes.

The yield curve for each month must be entered in columns D through M. The year and month are shown in columns A and B.

Historical yield curves can be obtained from the US Treasury at http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield_historical_main.shtml. That site provides daily yield curves. The curve to enter into this workbook for any month is the last daily yield curve from the website for that month.
The “Test Shocks” tab
The test referred to in the tab name is the Stochastic Exclusion Test. The 16 scenarios in that test are defined by feeding specified “random” shocks into the scenario generator. This tab shows the shocks that are used to define each scenario.

There are two buttons at the top of the page, one to “Show Shocks” and one to “Clear Shocks.” The workbook is distributed with none of the shocks actually shown on this tab because that reduces the file size by a significant amount. To show the shocks, click the “Show Shocks” button. To clear them again, press the “Clear Shocks” button.

Notes on how the shocks are used to define the stochastic exclusion test scenarios
The interest rate generator uses three shocks for each month, one to the long rate, one to the difference between long and short rates, and one to the volatility. The shocks to the long rate and to the difference between long and short rates are shown under the column headings of “LongInt” and “IntDiff” respectively. For purposes of the stochastic exclusion test scenarios the third shock, that to the volatility level, is always zero.

The equity fund scenario generator uses two shocks for each time period, one to the return level and one to the volatility. Just as with the interest rate generator, the shocks to volatility are zero for purposes of the stochastic exclusion test scenarios. The shocks to the return level are shown under the column heading “Equity.”