American Academy of Actuaries

November 22, 2023

Ms. Rachel Hemphill, Chair, Life Actuarial (A) Task Force (LATF)
Mr. Philip Barlow, Chair, Life Risk-Based Capital (E) Working Group (Life RBC)
Mr. Mike Yanacheak, Chair, Generator of Economic Scenarios (E/A) Subgroup (GOES Subgroup)
National Association of Insurance Commissioners (NAIC)

Dear Ms. Hemphill, Mr. Barlow, and Mr. Yanacheak,

The Economic Scenario Generator Subcommittee (ESGS) of the American Academy of Actuaries'[1] (the Academy) appreciates the ongoing opportunity to present proposals for stylized facts and acceptance criteria related to LATF's Generator of Economic Scenarios (GOES) project. These comments offer feedback and potential criteria related to Gross Wealth Factors (GWF) for equity index returns.

Executive Summary

In response to LATF's request, the ESGS has analyzed the forward-looking GWFs for S&P 500 total returns were previously developed by the Academy and proposed to LATF in 2005 and, using more recent historical data on S&P 500 total returns, developed the following updates.

In the course of this work, the ESGS found that equity returns can be effectively modeled using a constant mean, with the excess of that constant mean over the long-term target for interest rates representing a premium in exchange for the risk of holding an equity index over Treasuries. This is an equity risk premium (ERP. It is important to understand the differences and advantages of using a constant mean equity return rather than a constant mean equity risk premium to model returns on equity indices. While using a constant mean ERP is suitable for some purposes, we did not find it suitable for purposes with longer horizons that rely on realistic tail distributions, such as statutory reserves and capital for long duration, market-sensitive life and annuity products.

The methodology used to update GWFs and the updated GWFs themselves is addressed below. While the ESGS did not achieve consensus around a single target level for long-term equity returns, we offer three different sets of updated GWFs for consideration: one where the mean equity return was left unconstrained (straight fit to historical data resulting in a mean of 11.64%); one where the mean equity returns was constrained to 8.75% (the level used for the 2005 GWFs); and one where the mean equity return was constrained to 10.00% (roughly in the middle of the other two).

[1] The American Academy of Actuaries is a 19,500-member professional association whose mission is to serve the public and the U.S. actuarial profession. For more than 50 years, the Academy has assisted public policymakers on all levels by providing leadership, objective expertise, and actuarial advice on risk and financial security issues. The Academy also sets qualification, practice, and professionalism standards for actuaries in the United States.

The relationship between equity returns and interest rates

After reviewing historical data and economic theory, the ESGS believes a suitable method for modeling equity returns is to use a constant mean. Such a method produces a moderately inverse relationship between equity returns and interest rates, which is supported by history. It results in a higher ERP when simulated interest rates are low and vice versa, as well as an average ERP over the long term that is the positive excess of the constant mean for equity returns over the long-term target for interest rates. We have included theoretical and empirical rationale for the above statement, discussing the differences and advantages of using a method that employs a constant mean equity return rather than a constant mean ERP.

In 2005, the Academy proposed a set of GWF for validating equity scenarios used to determine capital for variable annuity products under C-3 Phase II. The ESGS has reviewed and updated those factors considering subsequent equity market performance and our previously proposed stylized facts for equity returns (see Appendix 1), which feature the concepts that:

- Cumulative equity returns tend to exceed the compounded risk-free rate (positive observed equity risk premium) over long time horizons, but over short time horizons the observed equity risk premium fluctuates due to several factors and can be negative." (Stylized Fact #2)[2]
- "Cumulative equity returns over long time horizons are not materially impacted by initial market conditions." (Stylized Fact #4)[3]

As discussed in academic, investment industry, and other papers, economic theory and empirical data suggest an ERP which is not constant over time[4] but instead one which varies over time in a countercyclical (inverse) manner,[5] expanding and contracting with business cycles. It may be higher in a recession[6] and when interest rates are low[7], and it may depend on other factors such as investor risk aversion and consumption preferences, inflation, quality and availability of earnings information, and government and monetary policy[8] (see figure 1).

Note that while the capital asset pricing model (CAPM) assumes a constant ERP, leading ERP research[5,6,7] indicates an ERP that changes over time. As a practical matter, market users of CAPM regularly update

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[2] For example, the Dividend Discount Model implies that equity valuations (i.e., present value of projected earnings) decrease when risk-free interest rates increase (i.e., an inverse relationship due to increased discounting when rates are high and vice versa). However, this inverse relationship can be magnified if the increase in interest rates is due to Fed policy to slow the economy, which will presumably also have a negative effect on projected earnings).

[3] This stylized fact is implicitly in the current AIRG as well as the 2005 C-3 Phase II GWF calibration standard.

[4] Expansionism: The Impact of the Fed's Monetary Regime on the Equity Risk Premium (Global Financial Data White Paper)

[5] The Equity Risk Premium: A Contextual Literature Review (CFA Institute Research Institute)

[6] Kroll Recommended U.S. Equity Risk Premium and Corresponding Risk-Free Rates to be Used in Computing Cost of Capital: January 2008 – Present

[7] The Equity Risk Premium: A Review of Models (Federal Reserve Bank of New York Staff Report No. 714)

[8] Equity Risk Premiums (ERP): Determinants, Estimation, and Implications—The 2021 Edition, with annual updates also available at Professor Damodaran's website

ERPs as a key input to the model[3], effectively treating the ERP as constant for the time horizon of interest. As such, the ERP is kept constant for the duration of the current period's cash flow projection and updated from one period to the next to reflect changes in the ERP over time.

Figure 1: Factors considered in Kroll's December 9, 2020, U.S. ERP Recommendation[9]
(relative change from March to November 2020)

FACTOR	CHANGE	EFFECT ON ERP
U.S. Equity Markets	▲	▼
Implied Equity Volatility	▼	▼
Corporate Spreads	▼	▼
Economic Policy Uncertainty (EPU) and Equity Uncertainty Indices	▼	▼
Historical Real GDP Growth and Forecasts	▼	▲
Unemployment Environment	▼	▲
Consumer Confidence	▼	▲
Business Confidence	▲	▼
Sovereign Credit Ratings	◀▶	◀▶
Damodaran Implied ERP Model	▼	▼
Default Spread Model	▼	▼

Under quantitative equity valuation models such as the discounted cash flow (DCF) or dividend discount model (DDM), the price of a stock is the present value of future dividends. All else being equal, a stock's price is expected to fall as interest rates increase. In addition, Federal Reserve monetary policy, which increases rates to slow the economy in the near-term while maintaining stability in the longer-term, may also reduce projected earnings and dividend growth (e.g., lower revenues, higher capital / borrowing costs), further adversely affecting stock prices in the near-term.

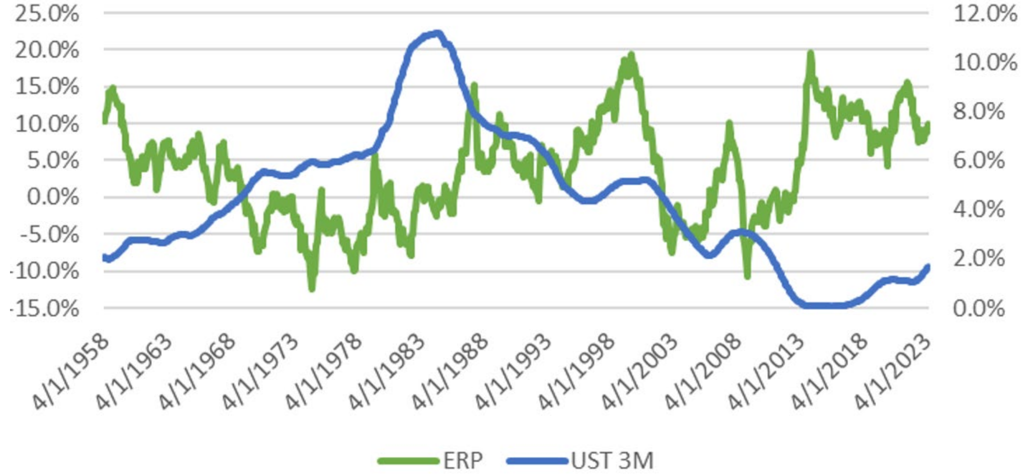
Changes in investor risk appetites as interest rates increase may also decrease the relative attractiveness of equities, such as the There Is No Alternative (TINA) effect widely cited in industry publications. TINA describes the preference for stocks and other risky assets during periods of near-zero interest rates, compared to a preference for Treasuries when risk-free rates on Treasuries are high and close to the risky returns on stocks.[8]

In addition to economic theory, it's also useful to consider empirical data and see how well a single-regime[9] or regime-switching model can be fit to empirical data. Exploratory graphical analysis of empirical data (see Figure 2) suggests an ERP that tends to be higher when interest rates are low and lower when interest rates

[9] Duff & Phelps Recommended U.S. Equity Risk Premium Decreased as COVID-19 Impact Recedes

are high (vs. randomly dispersed around a constant level).

Figure 2: 60-month rolling average: S&P 500 ERP vs. 3M UST



Statistical tests based on simple regression (i.e., a single regime) models, such as the 2022 Blitz paper[10], also tend to reject the hypothesis that a higher risk-free return implies higher total average stock returns. Rather, they show expected stock returns appear to be inversely related to the level of the risk-free return.

In addition to using single regime models, historical data may also be analyzed using more complex models that allow for the possibility of three types of regimes:

- Expected equity returns increase 1-for-1 as interest rates increase (expected ERP is constant)
- Expected equity returns remain constant as interest rates increase (expected ERP varies inversely with rates)
- Expected equity returns decrease 1-for-1 as interest rates increase (expected ERP has a strongly inverse relationship with rates).

In all three regimes, random variations result in a distribution of ERPs and returns around the expected values.

The ESGS's maximum likelihood estimation of such a model using historical S&P 500 total returns suggests the S&P 500 tends to move in the same direction as interest rates (a constant ERP relationship where equity returns increase as interest rates increase) about 10% of the time and in the opposite direction as interest rates (a strongly inverse ERP relationship where equity returns decrease as interest rates increase) about 30% of the time. However, we also found that 60% of the time equity returns tend not to move with interest rates but stay centered around a *constant* mean equity return, albeit a mean that is higher than average interest rate levels. This reflects an ERP that varies moderately inversely with interest rates, especially when interest rates are less volatile (see Figures 2 and 3). As with the 2022 Blitz paper, these findings suggest rejecting a constant mean ERP relationship where equity returns increase as interest rates increase and vice versa.

Hypothesis testing of different relationships between interest rates and equity returns based on monthly

[10] Expected Stock Returns When Interest Rates Are Low (Blitz, 2022)

historical data from April 1953 to December 2020 indicates neither a constant mean ERP relationship nor a constant mean equity return relationship can be rejected in the middle eight deciles of the distribution of risk-free interest rates (see Table 1). However, at the top and bottom deciles, i.e., 3M UST yield below 15bps and above 8.33%, the constant mean ERP relationship ought to be rejected with p-values at a 0.3% and 1.6% significance level respectively. Note that this analysis accounts for the underlying volatility of the equity returns. Alternatively, unconstrained regression of the ERP relationship in the data produces a much higher significance level of 8.2% and 35.7% in the bottom and top deciles, pointing to an inverse relationship between risk free rates and equity returns, a similar conclusion in the 2022 Blitz paper. Given the purpose at hand, modeling cash flows for the determination of statutory capital, it is important to reflect a plausible relationship between interest rates and equity returns in low and high tails which are likely to drive the total asset requirement (TAR) upon which capital is based (see Appendix 4 for additional details on this analysis).

Figure 3: Historical frequency of degree of relationship between monthly S&P price index returns and short-term interest rates observed from 1953.04 to 2020.12

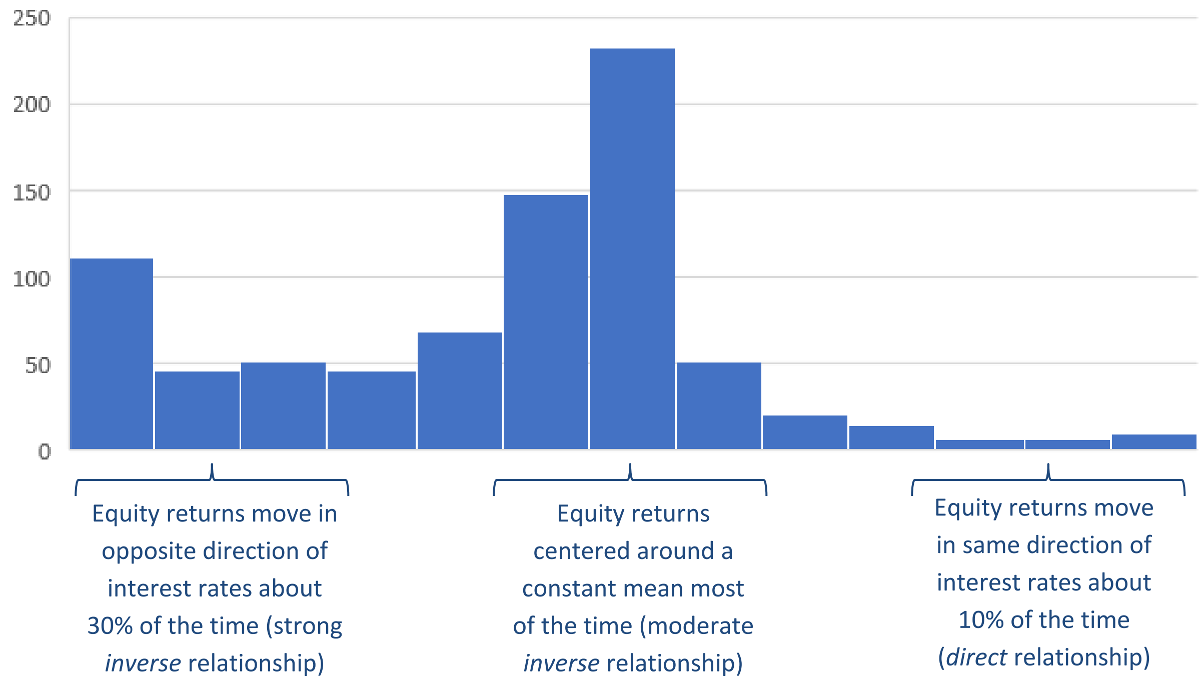


Table 1: p-values from constrained and unconstrained regression fits to monthly historical S&P 500 data from 1953.04 to 2020.12.				
Deciles of 3M UST Yield	**Average 3M UST Yield**	**Regression constrained to constant mean ERP**	**Regression constrained to constant mean Equity Return**	**Regression unconstrained**
0-10	0.1%	0.3%	2.4%	8.2%
10-20	0.8%	29.5%	50.0%	34.6%
20-30	1.9%	40.1%	42.5%	30.5%
30-40	2.8%	29.6%	20.8%	15.5%
40-50	3.7%	20.5%	16.1%	13.5%
50-60	4.6%	43.9%	46.2%	48.1%
60-70	5.2%	21.6%	17.1%	14.1%
70-80	6.0%	35.9%	45.6%	46.8%
80-90	7.5%	39.9%	41.7%	28.8%
90-100	10.7%	1.6%	12.6%	35.7%
Mean ERP =		***Constant***	***Constant - (1.00 × Risk Free Rate)***	***Constant - (1.74 × Risk Free Rate)***
Mean Equity Return =		***Risk Free Rate + Constant***	***Constant***	***Constant - (0.74 × Risk Free Rate)***

After reviewing historical data and economic theory, a suitable way to model an ERP that exhibits a moderately inverse relationship with interest rates uses a constant mean for simulated equity returns, resulting in a higher ERP when simulated interest rates are low and vice versa. It also uses an average ERP over the long term that is the positive excess of the long-term target for equity returns over the long-term target for interest rates. Under such an approach, the mean equity return not only stays constant throughout the projection, but also generally stays constant from one period to the next.[11] However, initial ERP levels would change from one period to the next as interest rates move. Under such an approach, steady state equity return targets should align with steady state interest rate levels, reflecting an appropriate long-term equity risk premium and sufficient dispersion of equity returns to allow for appropriate joint probabilities of low/high equity returns and low/high interest rates.

While some applications model the ERP as varying around a constant expected mean, such as GEMS where equity returns increase with interest rates and vice versa, this modeling simplification is less aligned with economic theory and historical data. It is more commonly and appropriately used for shorter-term projections of real-world scenarios, particularly when the computationally efficient generation of "nested" risk neutral scenarios is a priority (Solvency II), strategic asset allocation where the focus is on the middle of the distribution, or non-equity sensitive liabilities (short-duration property and casualty liabilities). It is not suitable for purposes with longer horizons that rely on realistic tail distributions, such as statutory reserves and capital for long duration or market-sensitive life and annuity products. An increasing return / constant expected ERP approach is more likely to distort reserve and capital levels by inadequately reflecting historically observed tail dynamics. It will also cause a distribution of equity returns that shifts from one period to the next in an unintuitive manner as interest rates move, potentially resulting in artificial and unintuitive volatility and/or distorting hedging and risk management decisions and costs. Workarounds that maintain this relationship in the ESG

[11] For example, major and long-lasting changes in fed policy or market dynamics may warrant a review and potential change to long term targets for equity returns.

model, but adjust the model's parameters whenever initial conditions change, may partially mitigate some of these impacts. It may also introduce other unintuitive dynamics, adding additional unnecessary operational and governance-type complexities.

In summary, the 4th stylized fact proposed for equity returns supports continued use of a static set of GWFs as criteria for equity returns, such as the set of GWFs proposed by the Academy in 2005 for validating equity scenarios used to determine capital for variable annuity products under C-3 Phase II. Although such criteria generally would not be expected to change over time, it is prudent to review them periodically, consider interim equity market experience, and determine if any deviations are material enough to warrant a change to the criteria. Under a constant mean equity return approach, there is still a relationship between interest rates and equity returns, but it is implicit in the long-term targets set for equity returns and interest rates. It is important to consider the reasonableness of the implied long-term ERP when reviewing and resetting those long-term targets.

Methodology and updates to criteria for equity GWFs

As in 2005, our updated analysis of equity GWFs was based on taking several model forms, consistent with our equity stylized facts and commonly used to model equity returns, and fitting them to monthly S&P 500 total returns from March 1957 through December 2022, including twenty additional years of relevant and credible data than the original 2005 analysis. This parallels the 2005 analysis done by the Academy to develop the first C-3 Phase II GWF criteria for equity returns. In that work, several similar equity model forms were used to inform what realistic equity return behavior could look like given the lack of historical data and limited availability of non-overlapping multi-year periods in the historical data set[12]. Appendix 2 details the model forms considered, fitted, and used to inform realistic equity market behavior and develop criteria for GWFs, such as Heston, SLV2, RSLN2, and Heston with Jump[13]. Tables at the end of Appendix 2 show annualized equity returns across the reference models for 1-year and 20-year horizons, compared to similar statistics for the AIRG and the GEMS Field Test (FT) #1a scenario sets, as well as to the distribution of rolling 1-year returns observed in history.

We initially fit reference models to history without constraining the mean equity return. Table 2 below shows the resulting unconstrained means for the various reference models are about 11.50%, or 2.75% higher than the constrained mean of 8.75% used in the Academy's 2005 analysis.

[12] As in 2005, the data period begins in the mid-1950s, when the Fed, securities regulation, and the S&P 500 began to resemble their modern-day counterparts (before the mid-1950s the S&P 500 was less of a broad large cap index, and securities regulation and fed/monetary policy functioned in a markedly different manner, e.g., lack of trade limits, significantly less national debt as a percentage of GDP). Note that reference models appropriately calibrated to this data period, which doesn't include the 1930s great depression, are still able to simulate large drops and sustained losses that are even worse than those experienced during the great depression.

[13] Heston with Jump is similar to GEMS, but without the constant expected ERP (increasing return) relationship.

Table 2:	30-year mean and median from unconstrained fits to 1957.03 to 2022.12	
Model	*Unconstrained mean*	*Unconstrained median*
*Heston**	11.47%	10.39%
SLV1	10.34%	9.13%
*SLV2**	11.37%	10.53%
SLV3	11.61%	10.81%
*RSLN2**	11.94%	10.48%
RSDD2	11.22%	10.34%
LN	11.68%	10.43%
*Jump**	11.80%	10.79%
AIRG†	8.81%	7.59%

** GWF tables below are based on the least binding percentile across these four selected reference models. The average mean across those four selected reference models is 11.64%.*

† The AIRG's constrained mean and median are provided for reference.

While the ESGS did not achieve consensus around a single target level for long-term equity returns, the group recommends the use of a *best-estimate* target, with appropriately disperse tails, rather than a target that is purposely set low or high. Since some products and risk management strategies perform worse when equity returns are low while others perform better, our recommendation is a distribution for stochastically modeled risk factors that is best estimate, with the CTE level used as the source of prescribed conservatism in statutory reserve and capital calculations. Although a long-term, best-estimate mean equity return target of 11.64% based on a pure fit to history may be too high, some felt a target of 8.75% (a 2.89% haircut to the historical fit of 11.64%) may be too low, indicating something in the middle may be more appropriate[14]. Regardless of the long-term target used for equity returns, it is important to periodically review the long term mean ERP implied by the long-term targets used for equity returns and interest rates for reasonableness. For purposes of this illustration, Tables 3.1, 3.2, and 3.3 below show the least binding GWF[15] across the four reference models using a constrained mean of 8.75%, a constrained mean of 10.00%, and an unconstrained mean of 11.64% (see Appendix 3 for a graphical representation of Table 3.2). We found that the least binding reference model was largely the same regardless of whether the mean was left unconstrained or constrained to 8.75% or 10%.

[14] Best-estimate forward-looking views on equity returns are often based on more than just historically observed equity returns, and also incorporate things such as expectations around real GDP growth, inflation, fed/monetary policy, industry surveys, etc.

[15] The least binding GWF across the four selected reference models is the maximum for low-return (left) tail percentiles and the minimum for high-return (right) tail percentiles (means and medians are shown for informational purposes and are *averages* across reference models).

Table 3.1:	Updated GWFs using a constrained mean return of 8.75%					Least binding reference model					
	Horizon (years)					*Horizon (years)*					
Percentile	*1*	*5*	*10*	*20*	*30*	*50*	*1*	*5*	*10*	*20*	*30*	*50*
Min	0.48	0.28	0.33	0.32	0.56	0.85	Heston	SLV2	SLV2	SLV2	SLV2	SLV2
1%	0.71	0.64	0.71	0.99	1.55	4.15	Heston	SLV2	SLV2	SLV2	SLV2	SLV2
5%	0.83	0.84	1.02	1.62	2.73	8.63	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
10%	0.89	0.98	1.22	2.10	3.74	12.78	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
15%	0.93	1.07	1.38	2.46	4.55	16.49	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
30%	1.02	1.28	1.76	3.41	6.84	27.56	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
Median	1.09	1.48	2.15	4.47	9.23	39.98	n/a	n/a	n/a	n/a	n/a	n/a
70%	1.17	1.73	2.70	6.14	13.50	62.71	Heston	SLV2	RSLN2	RSLN2	RSLN2	RSLN2
85%	1.24	1.97	3.27	8.41	20.39	112.78	SLV2	SLV2	SLV2	SLV2	SLV2	RSLN2
90%	1.28	2.09	3.58	9.59	23.93	142.63	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
95%	1.33	2.28	4.08	11.43	30.68	195.72	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
99%	1.42	2.67	5.10	15.83	45.17	333.02	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
Max	1.67	3.75	8.01	29.20	99.48	1019.62	SLV2	Jump	SLV2	SLV2	SLV2	SLV2
Mean	1.09	1.52	2.31	5.38	12.38	65.77	n/a	n/a	n/a	n/a	n/a	n/a

Table 3.2:	Updated GWFs using a constrained mean return of 10.00%					Least binding reference model					
	Horizon (years)					*Horizon (years)*					
Percentile	*1*	*5*	*10*	*20*	*30*	*50*	*1*	*5*	*10*	*20*	*30*	*50*
Min	0.49	0.29	0.36	0.40	0.80	1.51	Heston	SLV2	SLV2	SLV2	SLV2	SLV2
1%	0.72	0.68	0.79	1.25	2.18	7.36	Heston	SLV2	SLV2	SLV2	SLV2	SLV2
5%	0.84	0.89	1.15	2.03	3.84	15.27	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
10%	0.90	1.04	1.37	2.64	5.27	22.62	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
15%	0.94	1.14	1.55	3.09	6.41	29.20	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
30%	1.03	1.36	1.97	4.29	9.64	48.80	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
Median	1.11	1.57	2.41	5.62	13.00	70.81	n/a	n/a	n/a	n/a	n/a	n/a
70%	1.18	1.83	3.03	7.72	19.03	111.04	Heston	SLV2	RSLN2	RSLN2	RSLN2	RSLN2
85%	1.26	2.08	3.67	10.57	28.73	199.71	SLV2	SLV2	SLV2	SLV2	SLV2	RSLN2
90%	1.29	2.21	4.02	12.05	33.72	252.57	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
95%	1.34	2.42	4.57	14.37	43.23	346.58	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
99%	1.44	2.83	5.71	19.90	63.64	589.72	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
Max	1.69	3.97	8.98	36.70	140.17	1805.56	SLV2	Jump	SLV2	SLV2	SLV2	SLV2
Mean	1.10	1.61	2.59	6.76	17.45	116.46	n/a	n/a	n/a	n/a	n/a	n/a

Table 3.3:	Update GWFs using an unconstrained mean return of 11.64%					Least binding reference model					
	Horizon (years)					*Horizon (years)*					
Percentile	*1*	*5*	*10*	*20*	*30*	*50*	*1*	*5*	*10*	*20*	*30*	*50*
Min	0.49	0.31	0.41	0.51	1.15	2.80	Heston	SLV2	SLV2	SLV2	SLV2	SLV2
1%	0.73	0.72	0.90	1.60	3.15	13.63	Heston	SLV2	SLV2	SLV2	SLV2	SLV2
5%	0.85	0.95	1.30	2.60	5.56	28.30	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
10%	0.92	1.11	1.55	3.37	7.63	41.92	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
15%	0.95	1.21	1.75	3.96	9.28	54.11	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
30%	1.04	1.44	2.23	5.52	13.96	90.53	SLV2	SLV2	SLV2	Jump	SLV2	Jump
Median	1.12	1.69	2.79	7.56	20.27	148.11	n/a	n/a	n/a	n/a	n/a	n/a
70%	1.20	1.95	3.43	10.18	29.42	238.65	Heston	SLV2	SLV2	SLV2	SLV2	Heston
85%	1.27	2.22	4.15	13.53	41.60	377.39	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
90%	1.31	2.35	4.55	15.42	48.82	468.01	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
95%	1.36	2.57	5.17	18.39	62.60	642.20	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
99%	1.46	3.01	6.46	25.47	92.14	1092.72	SLV2	SLV2	SLV2	SLV2	SLV2	SLV2
Max	1.71	4.31	10.16	46.96	202.94	3345.63	SLV2	Jump	SLV2	SLV2	SLV2	SLV2
Mean	1.12	1.73	3.01	9.10	27.28	245.63	n/a	n/a	n/a	n/a	n/a	n/a

Tables 4.0, 4.1, 4.2, and 4.3 compare the updated GWFs to the prior set of C-3 Phase II GWFs[16]. Table 4.1 shows that when using the same constrained mean equity return of 8.75%, considering 20 years of additional data does not result in significantly different GWFs[17]. However, the updated GWFs in the above tables are now available for longer horizons and additional percentiles than the prior set of C-3 Phase II GWFs, given the expanded use of economic scenarios to for reserves, longer duration life insurance liabilities (e.g., VM-20 liabilities tend to be of longer duration than VM-21 liabilities), and the adoption of more extreme tail CTE levels in capital requirements (e.g., CTE98 vs. CTE90).

[16] The 2005 C-3 Phase II GWF calibration standard was based on an SLV model fit to S&P 500 total returns from 1955.12 to 2003.12, with the mean constrained to 8.75%, and further adjustments made to accommodate a wider range of reasonably fit ESG model forms (i.e., LN, RSLN). Note that the 2005 standard had criteria for percentiles of 2.5% and 97.5%, whereas updated GWFs have criteria for percentiles further in the tail at 1% and 99%.

[17] This is consistent with the last 20 years having both the 2008-2009 financial crisis as well as periods of very favorable returns. Note that the Academy's 2012 analysis suggested that the 2005 set of C-3 Phase II GWF criteria allowed the volatility and returns seen during the 2008-2009 financial crisis and did not warrant an update at that time.

Tables 4.0, 4.1, 4.2, and 4.3: Updated gross wealth factors compared to prior C3P2 calibration standard

Table 4.0:	Prior C3P2 GWF calibration standard (mean return constrained to 8.75%)			
	Horizon (years)			
Percentile	*1*	*5*	*10*	*20*
2.5%	0.78	0.72	0.79	n/a
5%	0.84	0.81	0.94	1.51
10%	0.90	0.94	1.16	2.10
90%	1.28	2.17	3.63	9.02
95%	1.35	2.45	4.36	11.70
97.5%	1.42	2.72	5.12	n/a

Note: The tables below only show the subset of updated GWFs that can be compared to the prior C3P2 calibration standard.

Table 4.1:	Proposed update to GWFs using constrained mean return of 8.75%				As a percentage of prior C3P2 calibration standard			
	Horizon (years)				*Horizon (years)*			
Percentile	*1*	*5*	*10*	*20*	*1*	*5*	*10*	*20*
5%	0.83	0.84	1.02	1.62	99%	104%	109%	107%
10%	0.89	0.98	1.22	2.10	99%	105%	105%	100%
90%	1.28	2.09	3.58	9.59	100%	96%	99%	106%
95%	1.33	2.28	4.08	11.43	98%	93%	94%	98%

Table 4.2:	Proposed update to GWFs using constrained mean return of 10.00%				As a percentage of prior C3P2 calibration standard			
	Horizon (years)				*Horizon (years)*			
Percentile	*1*	*5*	*10*	*20*	*1*	*5*	*10*	*20*
5%	0.84	0.89	1.15	2.03	100%	110%	122%	135%
10%	0.90	1.04	1.37	2.64	100%	111%	118%	126%
90%	1.29	2.21	4.02	12.05	101%	102%	111%	134%
95%	1.34	2.42	4.57	14.37	100%	99%	105%	123%

Table 4.3:	Proposed update to GWFs using unconstrained mean return (11.64%)				As a percentage of prior C3P2 calibration standard			
	Horizon (years)				*Horizon (years)*			
Percentile	*1*	*5*	*10*	*20*	*1*	*5*	*10*	*20*
5%	0.85	0.95	1.30	2.60	101%	117%	138%	172%
10%	0.92	1.11	1.55	3.37	102%	118%	134%	161%
90%	1.31	2.35	4.55	15.42	102%	108%	125%	171%
95%	1.36	2.57	5.17	18.39	101%	105%	119%	157%

Depending on the desired long-term mean equity return target, tables 3.1, 3.2, or 3.3 can be considered as a potential update to the 2005 set of C-3 Phase II GWF criteria[18]. Such criteria are applied to a scenario set by checking if the corresponding percentiles from the scenario set are more extreme than the criteria. Although such a one-way check helps ensure the distribution of GWFs is plausibly more extreme than history, when applying the criteria, it may also be useful to note the magnitude of the differences between criteria and the corresponding percentiles from the scenarios set. Differences that are too large may indicate a distribution of GWFs that is unreasonably more extreme than history.

In summary, after reviewing economic theory and a data period that includes 20 additional years of experience, the Academy supports a modeled ERP that is implicit in the long-term best-estimate targets (i.e., means) used for interest rates and long-term equity returns, resulting in a distribution of equity returns that does not change from period to period, instead changing returns only when long-term targets are revised. GWFs reflecting the additional 20 years of experience (more extreme percentiles of 1% and 99%, and longer horizons of 30 and 50 years) have been illustrated using a constrained mean of 8.75%, a constrained mean of 10.00%, and an unconstrained mean of 11.64%.

The Academy looks forward to our continued collaboration with regulators on the GWFs illustrated, including reasonable "best-estimate" ranges for long-term targets on interest rates, equity returns, and ERP. We look forward to providing additional comments on the overall application of a complete set of stylized facts and acceptance criteria, as well as proposal for "quadrant" criteria related to the tails of the joint distribution of equity returns and interest rates, such as the prevalence of low interest rates and low equity returns, and high interest rates and low equity returns[19]. Please direct any questions to Amanda Barry-Moilanen, life policy analyst at barrymoilanen@academy.org.

Sincerely,

Jason Kehrberg
Chair, Economic Scenario Generator Subcommittee

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[18] The Min, Max, Median and Mean, as well as the Least binding reference model, are shown for reference and not specifically for inclusion in an updated set of GWF criteria for equity returns. If a smaller set of GWF criteria is desired, the Academy would recommend keeping the more extreme percentiles of 1%, 5%, 95%, and 99%, along with the somewhat less extreme percentiles of 15% and 85% (i.e., dropping percentiles of 10%, 30%, 70%, and 90%). Note that these updated GWF criteria were developed for use on sets of 10,000 scenarios but could be considered for use on sets with fewer scenarios. Also note that these updated GWF criteria have been expressed in terms of gross wealth factors ("GWFs") but can also be expressed in terms of geometric average returns ("GAVGs") by using the formula: *GAVG = GWF^(1/horizon) - 1*, where *horizon* is in years.

[19] LATF has previously requested the ESGS develop a proposal for such "quadrant" criteria.

Appendix 1—Stylized facts for Equity Returns (presented by Academy to LATF on 9/29/22)

1. Equity indices (indeed, all asset classes) tend to exhibit **consistent risk/reward relationships** over **long** time horizons.
2. Cumulative equity returns tend to exceed the compounded risk-free rate (positive observed **equity risk premium**) over long time horizons, but over short time horizons the observed equity risk premium fluctuates due to several factors and can be negative.
3. Equities **fluctuate between bull and bear markets** (bubbles tend to burst)—markets can experience significant losses but eventually tend to **move back into positive territory** (negative cumulative equity returns become less likely over longer time horizons).
4. Cumulative equity returns ***over long time horizons*** **are not materially impacted by initial market conditions**.
5. The **volatility of equity returns varies over time but has a strong tendency to revert to normative levels**. This allows for both extreme gains and extreme losses over short time periods (i.e., the distribution has fat tails, or **positive kurtosis**). Furthermore, the **volatility of equity returns is higher in bear markets**. This increases the probability of extreme losses relative to extreme gains (i.e., the distribution has a longer left tail, or **negative skewness**).
6. Equity markets contain **pathwise dynamics** over long time horizons that aren't present in the distribution of single-period returns. Future equity scenarios should have reasonable distributions of cumulative equity returns over long time horizons (e.g., 10, 20, 30 years), especially since these distributions are key to the performance of long-duration life and annuity products.
7. Future equity scenarios should include events that are **plausibly more extreme than history**.
8. Equity returns have both a **price and dividend component**, and they behave differently—dividend returns tend to be more stable than price returns.
9. Returns between different equity indices **are generally positively correlated** over **long** time horizons. This correlation may increase sharply in bear markets, but it tends to revert to normative levels in a short period of time.

Appendix 2—Detail on reference models considered for equity returns

A. Heston with Jumps ("Jump")[20]

- Process specified for discrete monthly timestep, $\Delta_t = 1/12$
- Stochastic equity variance follows Heston/CIR:

(a) $$v_t = \max\left[\,\tau^2(1-\zeta) + v_{t-1}\zeta + \sigma\sqrt{\tau^2/_{2\varphi}\,(1-\zeta)^2 + {}^{v_{t-1}}/_{\varphi}\,(\zeta-\zeta^2)}\,Z_t^v,\ v_{min}\right]$$

where:

  - τ^2, φ *and* σ are the steady state target, reversion speed, and monthly diffusion coefficient of the monthly variance process
  - $\zeta = e^{-\varphi}$
  - $Z_t^v \sim N(0,1)$
  - v_{min} is a suitably small floor to ensure stability of the variance process
  - v_0 is set to $v_0 = initial\ volatility^2$

- Periodic log-return follows:

(b) $$lr_t = (A + (C - .5)v_{t-1})\,\Delta_t + \sqrt{v_{t-1}\,\Delta_t}\,Z_t^{lr} - \lambda_t m + N_t\mu_j + \sigma_j\sqrt{N_t}Z_t^j$$

where:

  - v_{t-1} is the beginning-of-the-period Heston variance defined above
  - $Z_t^{lr} \sim N(0,1)$, where $\langle\, Z_t^{lr},\, Z_t^v\rangle = \rho$ is the correlation parameter between variance and log-return process
  - $\lambda_t = v_{t-1}\,\lambda_1\,\Delta_t$ is the intensity of Poisson counting process tied to the beginning-of-period Heston variance v_{t-1}
  - $N_t \sim Poisson(\lambda_t)$ is a Poisson random variable that is un-correlated with Heston variance and the core return process
  - $m = e^{\mu_j + .5\sigma_j^2} - 1$, with μ_j and σ_j as mean and volatility of the variable jump γ
  - Z_t^j is a standard normal variate that drives the log-normal jump diffusion of γ

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[20] Note that GEMS employs a Heston with Jumps model for equity returns.

B. Heston

- Process specified for discrete monthly timestep, $\Delta_t = 1/12$
- Stochastic equity variance follows Heston/CIR:

(a) $$v_t = \max\left[\,\tau^2(1-\zeta) + v_{t-1}\zeta + \sigma\sqrt{{\tau^2}/{2\varphi}\,(1-\zeta)^2 + {v_{t-1}}/{\varphi}\,(\zeta-\zeta^2)}\;Z_t^v,\; v_{min}\right]$$

where:

- τ^2, φ *and* σ are the steady-state target, monthly reversion speed, and diffusion coefficient of the monthly variance process
- $\zeta = e^{-\varphi}$
- $Z_t^v \sim N(0,1)$
- v_{min} is a suitably small floor to ensure stability of the variance process
- v_0 is set to $v_0 = initial\ volatility^2$

- Periodic log-return follows:

(b) $$lr_t = (\mu_0 - .5v_{t-1})\,\Delta_t + \sqrt{v_{t-1}\,\Delta_t}\;Z_t^{lr}$$

where:

- v_{t-1} is the beginning-of-the-period Heston variance defined above
- $Z_t^{lr} \sim N(0,1)$, where $\langle\, Z_t^{lr},\, Z_t^v \rangle = \rho$ is the correlation parameter between variance and log-return process

C. Stochastic Log Volatility ("SLV")[21]

- Process specified for discrete monthly timestep, $\Delta_t = 1/12$
- Stochastic equity log-volatility follows OU process:

  (a) $lv_t = \min\left[\varphi\tau + (1-\varphi)lv_{t-1},\ log(SoftMaxVol)\right] + \sigma\, Z_t^{lv}$

  where:

  - τ, φ *and* σ are the steady-state target, reversion speed, and monthly diffusion coefficient of the monthly log-volatility process
  - $Z_t^{v} \sim N(0,1)$
  - $vol_t = \max\left[\min\left(e^{lv_t}, MaxVol\right),\ MinVol\right]$
  - lv_0 is set to $lv_0 = log(initial\ volatility)$

- Periodic log-return follows:

  (b) $lr_t = (A + B\, vol_t + C\, vol_t^2)\, \Delta_t + vol_t \sqrt{\Delta_t}\, Z_t^{lr}$

  where:

  v_{t-1} is the beginning-of-the-period Heston variance defined above

  $Z_t^{lr} \sim N(0,1)$, where $\langle Z_t^{lr},\ Z_t^{v} \rangle = \rho$ is the correlation parameter between log-volatility and log-return process

[21] Note that the AIRG employs an SLV model for equity returns.

D. Lognormal (“LN”)[22]

- Process specified for discrete monthly timestep, $\Delta_t = 1/12$

  (a) $lr_{1,t} = \mu\Delta_t + \sigma\sqrt{\Delta_t}Z_t$

  where:

  - $Z_t \sim N(0,1)$

[22] The LN model was included in our analysis due to its simple nature and historical significance, however over longer time horizons it does not meet all our stylized facts for equity returns and so was not used to derive the updated tables of least binding GWFs. In particular, the LN model lacks negative skew, and doesn’t exhibit excess kurtosis over longer time horizons.

E. Regime-Switching Lognormal Model for 2 regimes (“RSLN2”)

- Process specified for discrete monthly timestep, $\Delta_t = 1/12$
- Periodic log-return for two regimes follows:

  (a) $lr_{1,t} = \mu_1 \Delta_t + \sigma_1 \sqrt{\Delta_t} Z_t$

  (b) $lr_{2,t} = \mu_2 \Delta_t + \sigma_2 \sqrt{\Delta_t} Z_t$

  where:

  - $Z_t \sim N(0,1)$
  - $p11, and\ p21$ are monthly transition probabilities indicating continuing in state 1, and migrating from state 2 to state 1 respectively.
  - For each scenario, the initial state is initialized to 1 if the scenario-specific $U(0,1) < p_1 = {p_{21}}/{p_{21} + p_{12}}$, and to state 2 otherwise.
  - Transition states are evolved using independent and identically distributed uniform variates compared to transition probabilities in the subsequent projection steps.

F. Regime-Switching Lognormal Model for 2 regimes with Draw Down ("RSDD2")[23]

- Process specified for discrete monthly timestep, $\Delta_t = 1/12$
- Periodic log-return for two regimes follows:

  (a) $lr_{1,t} = \mu_1 \Delta_t + \varphi_1 DD_t + \sigma_1 \sqrt{\Delta_t} Z_t$

  (b) $lr_{2,t} = \mu_2 \Delta_t + \varphi_2 DD_t + \sigma_2 \sqrt{\Delta_t} Z_t$

  where:
  - $Z_t \sim N(0,1)$
  - Draw Down at each projection point, t, is defined as $DD_t = min(0,\ DD_{t-1} + lr_{t-1})$ and is initialized with $DD_1 = 0$.
  - $p11, and\ p21$ are monthly transition probabilities indicating continuing in state 1 and migrating from state 2 to state 1 respectively.
  - For each scenario, the initial state is initialized to 1 if the scenario-specific $U(0,1) < p_1 = {}^{p_{21}}/_{p_{21} + p_{12}}$, and to state 2 otherwise.
  - Transition states are evolved using independent and identically distributed uniform variates compared to transition probabilities in the subsequent projection steps.

[23] The RSDD2 model was included in our analysis because it met our stylized facts for equity returns, but it was not used to derive the updated tables of least binding GWFs. RSDD2 has theoretical and empirical support but it is sensitive to the data period used and risks understating extreme events like the great depression. Had RSDD2 been used to derive the updated tables of least binding GWFs, the distribution of GWFs would have been narrower, i.e., *less* constraining.

G. Parameters and sum of log likelihood for reference models fit using unconstrained Maximum Likelihood Estimation (MLE) based on S&P 500 monthly returns from 1957.03 to 2022.12

	Heston	Jump	SLV	AIRG
tau	0.14694	0.14242	0.13076	0.12515
phi	0.09317	0.08436	0.09871	0.35229
sigma	0.04130	0.03805	0.16559	0.32645
A	0.10844	0.10886	0.09904	0.05500
B	0.00000	0.00000	0.00000	0.56000
C	0.00000	0.13580	2.45530	-0.90000
correlation (skew)	-0.54794	-0.58593	-0.68936	-0.24880
initial vol	0.14467	0.14242	0.15010	0.14760
min vol	0.03000	0.03000	0.03000	0.03050
soft max vol	0.30000	0.30000	0.30000	0.30000
max vol	0.35000	0.35000	0.35000	0.79880
mu_jump	0	-0.14740	0	0
sigma_jump	0	0.07000	0	0
lambda_jump	0	2.51937	0	0
MLE: Sum of LL	***1,430***	***1,435***	***1,447***	***1,418***

	RSLN2	RSDD2	LN
p11	0.93540	0.94077	1.00000
p21	0.10313	0.17652	0.00000
mu1	0.16570	0.13209	0.09910
mu2	-0.00720	-0.15209	0.00000
sigma1	0.09901	0.10749	0.14835
sigma2	0.20042	0.21292	0.10000
phi1	0	-0.06935	0
phi2	0	-0.00317	0
MLE: Sum of LL	***1,413***	***1,421***	***1,418***

H. Log return statistics for reference models fit using unconstrained Maximum Likelihood Estimation (MLE) based on S&P 500 monthly returns from 1957.03 to 2022.12

Monthly Log Return Statistics (first 600 months, 10k scenarios)								
	Heston	**Jump**	**SLV**	**RSLN2**	**RSDD2**	**LN**	**AIRG**	**History**
mean	0.81%	0.84%	0.83%	0.83%	0.83%	0.83%	0.61%	0.83%
st. dev.	4.25%	4.26%	4.28%	4.29%	4.29%	4.28%	4.36%	4.28%
skew	-0.03	-0.32	-0.69	-0.33	-0.56	0.00	-0.67	-0.67
kurt	4.25	5.79	5.47	4.39	4.63	3.00	7.00	5.32

Annual Log Return Statistics (first 50 years, 10k scenarios)								
	Heston	**Jump**	**SLV**	**RSLN2**	**RSDD2**	**LN**	**AIRG**	**History**
mean	9.78%	10.07%	9.93%	9.79%	9.90%	9.90%	7.31%	10.16%
st. dev.	14.92%	14.88%	14.14%	15.96%	15.72%	14.84%	15.13%	15.41%
skew	-0.58	-0.66	-0.74	-0.53	-0.66	0.00	-0.41	-0.90
kurt	4.20	4.41	4.77	3.73	4.36	3.01	4.12	4.54
Average Annual Return (30 yrs)	11.47%	11.80%	11.37%	11.94%	11.22%	11.68%	8.81%	n/a
Amount over AIRG	2.66%	2.99%	2.56%	3.13%	2.42%	2.87%	0.00%	n/a

I. Distribution of 1-yr and 20-yr returns for reference models fit using Maximum Likelihood Estimation (MLE) based on S&P 500 monthly returns from 1957.03 to 2022.12

The tables below show annualized equity returns across the reference models for 1-year and 20-year horizons compared to similar statistics for the AIRG and the GEMS Field Test (FT) #1a[24] scenario sets as well as to the distribution of rolling 1-year returns observed in history.

All reference models except for Lognormal (LN) allow for explicit return/volatility (negative skew) and produce scenarios with 1-year losses and gains that exceed the worst seen in history since 1957.03. Furthermore, when mean equity returns are constrained to 8.75% all reference models produce scenarios with negative returns over the first 20 years, an event not seen in history even including the great depression, and approximately 1% of scenarios experience negative returns over the first 20 years.

GEMS FT #1a shows a much lower median of 6.7% over the first year due to equity returns in that scenario set keying off the short interest rate which starts near 0% in that scenario set, while its first percentile is like the reference models (other than LN), i.e., reference models that model equity returns using a constant mean are able to achieve rates as low as GEMS FT #1a, even when starting short rates are near 0%. Over the first 20 years, GEMS FT #1a (which was calibrated to align with the AIRG 30-year GWF over the long-term) shows a median of 7.2% that is similar to the reference models when the mean equity return is constrained to 8.75% (first percentiles are also similar, as was the case over the first year).

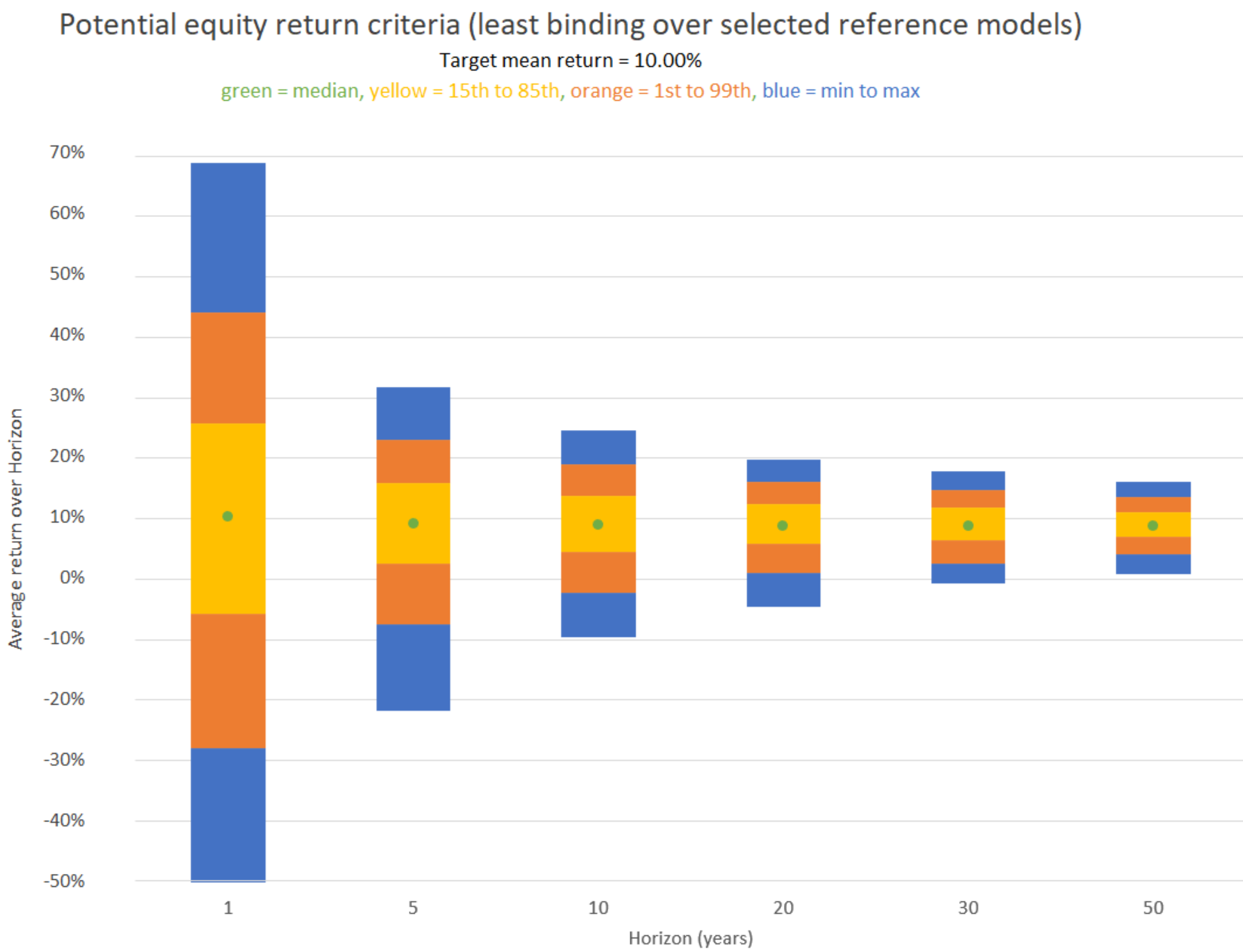
Distribution of 1-yr Return (Historical MLE, unconstrained mean)								**GEMS**	
	Heston	**Jump**	**SLV**	**RSLN2**	**RSDD2**	**LN**	**AIRG**	**FT #1a**	**History**
min	-50.7%	-51.5%	-52.3%	-58.3%	-51.8%	-41.9%	-59.5%	-49.7%	-43.3%
1	-26.9%	-27.7%	-28.0%	-29.2%	-31.1%	-22.1%	-28.5%	-29.2%	-33.2%
5	-15.1%	-15.0%	-14.8%	-17.6%	-18.2%	-13.6%	-17.3%	-18.2%	-15.2%
15	-5.5%	-4.8%	-4.5%	-5.7%	-6.4%	-5.4%	-7.7%	-8.7%	-5.0%
30	3.2%	3.9%	4.3%	3.8%	2.1%	2.3%	0.5%	-0.9%	5.3%
50	11.7%	12.5%	12.3%	12.4%	9.7%	10.8%	8.2%	6.7%	13.3%
70	19.7%	20.5%	20.2%	20.7%	17.6%	19.6%	16.4%	14.1%	19.9%
85	28.2%	28.2%	27.3%	28.8%	25.2%	28.8%	25.3%	21.7%	27.9%
95	38.1%	37.3%	36.1%	39.6%	34.8%	40.8%	36.5%	30.6%	37.8%
99	49.5%	48.5%	45.9%	53.0%	47.0%	56.1%	51.6%	41.0%	48.2%
max	82.9%	72.1%	71.0%	96.1%	78.9%	89.3%	85.2%	80.9%	61.1%

[24] Note that GEMS Field Test scenario (FT) set #2a would produce higher GWFs than GEMS FT #1a. This is because those two scenario sets model equity returns as a constant mean equity risk premium over the short rate, and starting short rates were significantly higher in GEMS FT #2a than GEMS FT #1a, especially for the one-year horizon.

Distribution of 20-yr Return (Historical MLE, unconstrained mean)								GEMS	
	Heston	Jump	SLV	RSLN2	RSDD2	LN	AIRG	FT #1a	History
min	-3.7%	-3.6%	-3.3%	-6.7%	-0.5%	-3.0%	-4.4%	-4.5%	n/a
1	1.2%	1.2%	2.4%	0.9%	3.7%	2.2%	-0.6%	-0.9%	n/a
5	4.0%	4.3%	4.9%	3.7%	5.5%	4.6%	1.7%	1.5%	n/a
15	6.5%	7.0%	7.1%	6.2%	7.3%	6.7%	3.9%	3.4%	n/a
30	8.5%	8.9%	8.9%	8.4%	8.7%	8.5%	5.8%	5.3%	n/a
50	10.5%	10.9%	10.6%	10.6%	10.3%	10.4%	7.7%	7.2%	n/a
70	12.4%	12.8%	12.3%	12.7%	11.9%	12.4%	9.5%	9.3%	n/a
85	14.2%	14.5%	13.9%	14.8%	13.6%	14.4%	11.4%	11.3%	n/a
95	16.3%	16.4%	15.7%	17.2%	15.6%	16.7%	13.7%	14.0%	n/a
99	18.4%	18.7%	17.6%	19.9%	17.8%	19.3%	16.2%	17.1%	n/a
max	22.1%	22.2%	21.2%	27.6%	23.1%	28.4%	20.3%	22.3%	n/a

Distribution of 20-yr Return (Historical MLE, mean constrained to 8.75%)								GEMS	
	Heston	Jump	SLV	RSLN2	RSDD2	LN	AIRG	FT #1a	History
min	-6.1%	-6.2%	-5.6%	-9.4%	-2.7%	-5.5%	-4.4%	-4.5%	n/a
1	-1.2%	-1.5%	0.0%	-2.0%	1.4%	-0.4%	-0.6%	-0.9%	n/a
5	1.5%	1.5%	2.4%	0.7%	3.1%	1.8%	1.7%	1.5%	n/a
15	3.9%	4.1%	4.6%	3.2%	4.9%	3.9%	3.9%	3.4%	n/a
30	5.8%	5.9%	6.3%	5.3%	6.3%	5.7%	5.8%	5.3%	n/a
50	7.8%	7.8%	8.0%	7.4%	7.9%	7.5%	7.7%	7.2%	n/a
70	9.7%	9.7%	9.7%	9.5%	9.4%	9.4%	9.5%	9.3%	n/a
85	11.4%	11.4%	11.2%	11.5%	11.0%	11.4%	11.4%	11.3%	n/a
95	13.4%	13.2%	13.0%	13.9%	13.0%	13.7%	13.7%	14.0%	n/a
99	15.6%	15.5%	14.8%	16.5%	15.1%	16.2%	16.2%	17.1%	n/a
max	19.2%	18.9%	18.4%	24.0%	20.3%	25.0%	20.3%	22.3%	n/a

Appendix 3—Graphical view of table 3.2 showing proposed criteria for selected percentiles based on a constrained mean equity return of 10.00%



Appendix 4—Additional analysis on the relationship between equity returns and interest rates

A. Summary of additional analysis

Historical Observations:

- Realized equity return (and ERP) are <u>*inversely related*</u> to the 3M UST rate in the top and bottom deciles, i.e., 17.6% equity return (17.6% ERP) when 3M UST rates are below 15 basis points, and 0.6% equity return (-9.9% ERP) when UST 3M rates are above 8.33%.
- The Fed's mandate includes balancing moderate inflation against employment and economic growth. Monetary tightening (e.g., when inflation is high) and easing (e.g., to stimulate the economy) rely on fundamental economic principles and contribute to the observed equity/rate dynamics of equity returns being <u>*inversely*</u> related to the level of 3M UST rates at tail levels.

GEMS assumed relationship:

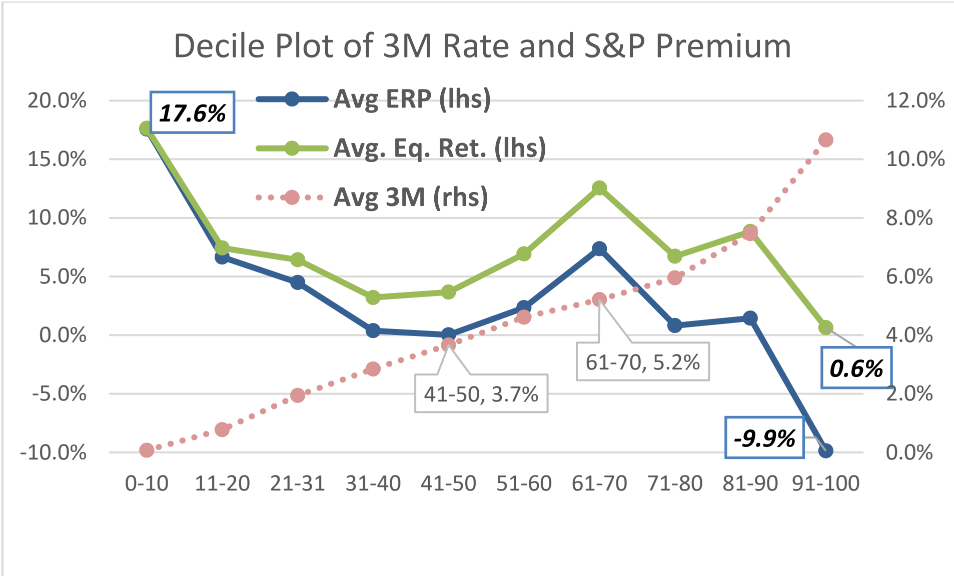
- GEMS assumes a constant expected ERP regardless of the level of rates, i.e., assumes expected equity return is an <u>*increasing*</u> function of the short (e.g., 3M) UST rate, which is opposite the historical relationship observed in the tails.
- Historical data suggests a constant ERP should be rejected at p-values of **0.3%** and **1.6%** in low- and high-rate deciles respectively. Low p-values indicate the model is not able to explain the realized equity return, even accounting for historical volatility.

AIRG assumed relationship:

- The AIRG model assumes a <u>*constant*</u> expected equity return, i.e., an ERP that decreases with the level of short (e.g., 3M) UST rate.
- While stylized, this assumption represents a middle ground between an inverse relationship seen in history and an increasing relationship embedded in GEMS.
- The AIRG model would be rejected at a p-value of 2.4% in the low-rate decile but shows p-values of 10%+ across the rest of the distribution.

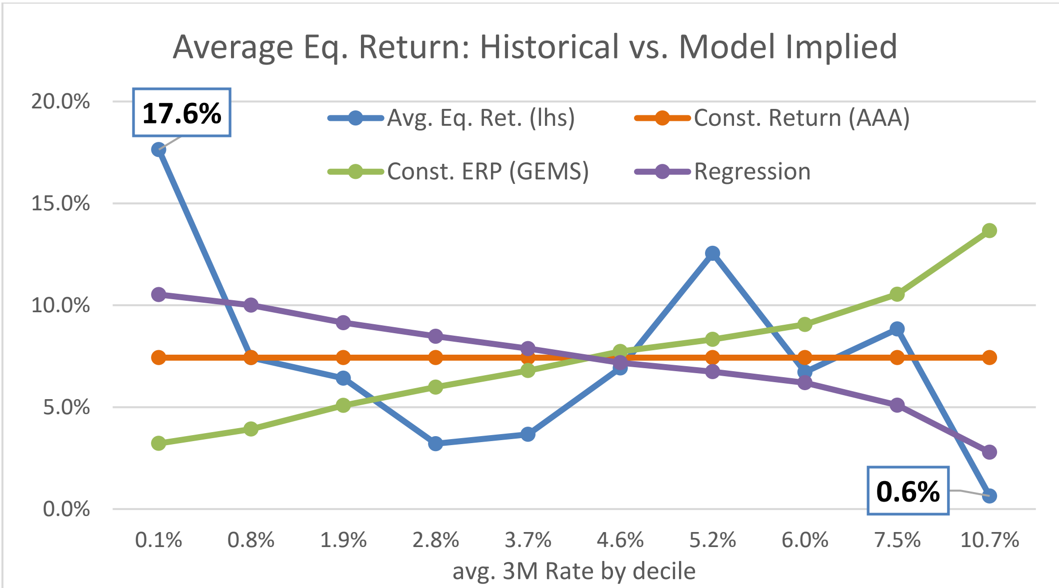
Both the AIRG (constant mean equity return) and GEMS (constant mean equity risk premium) assume ERP dynamics which are stylized simplifications of the complete historical record. Both may adequately account for historical equity returns in the middle 80% of the distribution and on average over the long term, but the constant mean equity return approach better accounts for the historical dynamics (and underlying Fed policy) observed in the tails (i.e., top and bottom deciles).

B. Realized Excess Equity Returns are Inversely Tied to Tail Rate Levels



- Based on monthly 3m treasury and S&P price index data from 4/1953 to 12/2020, covering 814 months, or 67.8yrs.
- Monthly ERP calculated as return on S&P less average 3M yield through the month, expressed on continuous/log basis.
- Each decile represents 81-82 monthly points, or 6.8yrs, where data was grouped by 3M rate.
- Average Eq. Return and ERP were then calculated for each decile, and annualized.
- Realized equity return and ERP are inversely related to 3M rate in the top and bottom deciles: 17.6% equity return (17.6% ERP) where rates are below 15bp, and 0.6% return (-9.9% ERP) when rates are above 8.33%.
- Historically, the 3M treasury rate is strongly tied to Fed Funds rate, which is typically targeted by the Fed to achieve its objectives under different market environments: Easing post 2008 Financial Crisis intended to stimulate economic growth and employment, resulted in near 0% short rates for most of the last 11 years. The policy, facilitating borrowing and spending, had contributed to growth in equities and outperformance over the money market.
- Fed tightening intended to control high inflation in 70s and 80s lead to double-digit short rates, especially in late 70s through early 80s. As intended, the policy of flooring borrowing rates at historically high levels stimulated savings while stifling inflation and economic growth, contributing to money-market outperformance vs. equities over an extended period.

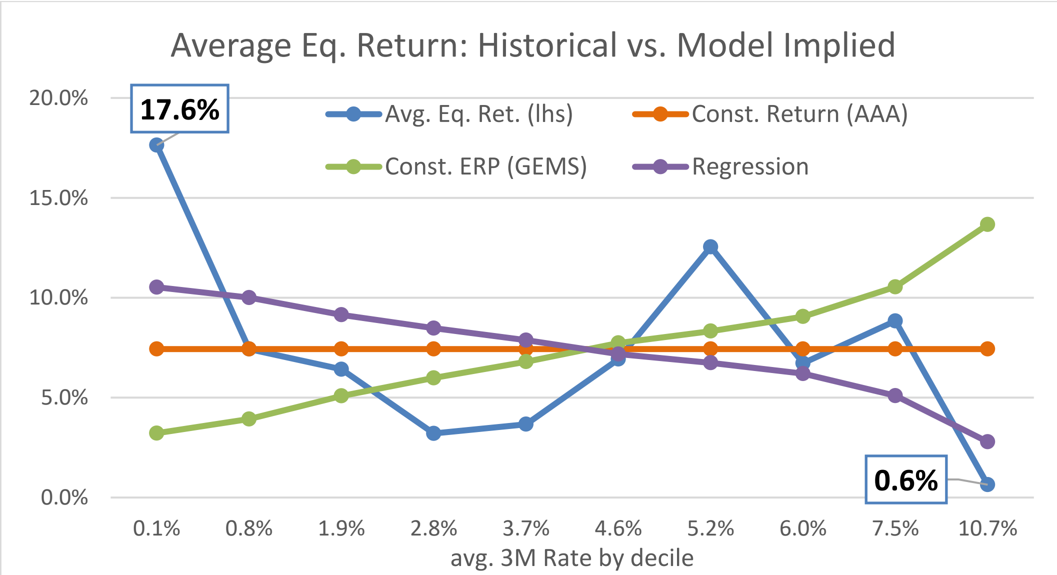
C. Simplified Modeling of Equity Return/ERP



	Avg. ERP (RF)	Avg. Return (RF)	Avg. Implied ERP	Avg. Implied Return
Constant Return (AIRG)	7.43% - RF	7.43%	3.15%	7.43%
Constant ERP (GEMS)	3.15%	RF + 3.15%	3.15%	7.43%
Regression Line	10.58% - 1.74 x RF	10.58% - .74 x RF	3.13%	7.41%

- Above assumes: **Equity Return = RF + ERP**, where RF is the return associated with risk free rate, and ERP is the Equity Risk Premium or excess earned over RF.
- Analysis considers constant ERP, constant Return, and ERP as a linear function of risk-free rate.
- While no considered approach perfectly captures historical data, a regression line showing inversely related equity return and risk-free rate, best aligns with the decile distribution.
- Constant (expected) Return (used in AIRG) implies ERP that is inversely related to risk-free rate.
- Constant (expected) ERP assumption, such as used in GEMS, results in increasing equity returns as a function of short rate – *directionally opposite* to what has been observed. Economic scenarios based on constant ERP would tend to:
  - Produce overly punitive equity returns in low-rate tail scenarios.
  - Understate the risk of adverse equity performance in high-rate tail scenarios.

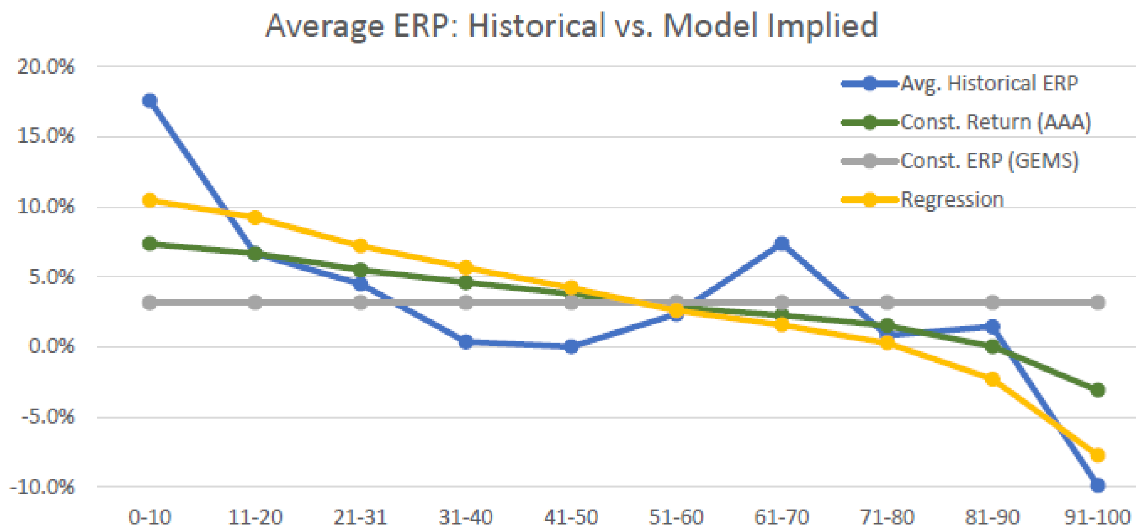
D. Historical Example: Fitting to January 2011 Treasury Curve



Deciles of 3M UST Yield	Average 3M UST Yield	Regression constrained to constant mean ERP	Regression constrained to constant mean Equity Return	Regression unconstrained
0-10	0.1%	0.3%	2.4%	8.2%
10-20	0.8%	29.5%	50.0%	34.6%
20-30	1.9%	40.1%	42.5%	30.5%
30-40	2.8%	29.6%	20.8%	15.5%
40-50	3.7%	20.5%	16.1%	13.5%
50-60	4.6%	43.9%	46.2%	48.1%
60-70	5.2%	21.6%	17.1%	14.1%
70-80	6.0%	35.9%	45.6%	46.8%
80-90	7.5%	39.9%	41.7%	28.8%
90-100	10.7%	1.6%	12.6%	35.7%
Mean ERP =		***Constant***	***Constant - (1.00 × Risk Free Rate)***	***Constant - (1.74 × Risk Free Rate)***
Mean Equity Return =		***Risk Free Rate + Constant***	***Constant***	***Constant - (0.74 × Risk Free Rate)***

- P-values were calculated to test the null hypothesis where average observed Equity Return within each decile was generated by each of the simplified ERP models.
- A small value of p, below a significance level (a popular choice is 5%) implies that the null hypothesis can be rejected with high confidence.
- Constant ERP produces very small p-values in the tails, especially in the first decile, where p = 0.3% implies that the model would produce average ERP/equity return > 17.6% over 82 months with only 0.3% probability.
- Constant Return assumption (used in AIRG) improves upon Constant ERP in the tails and is better capable of generating scenarios that reflect historical macro-economic/fed policy interactions.

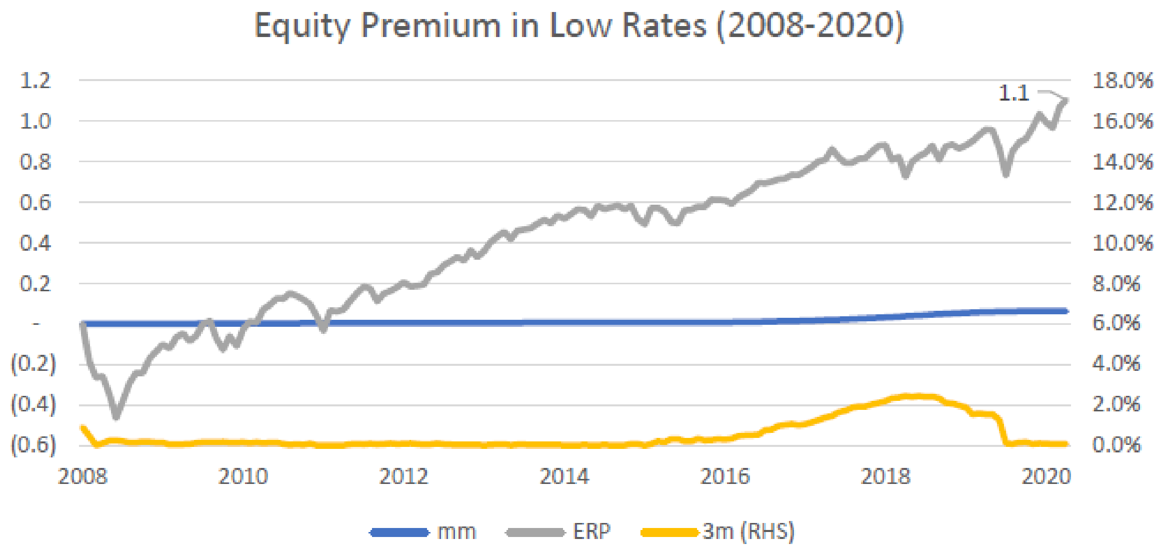
E. Simplified Modeling of ERP

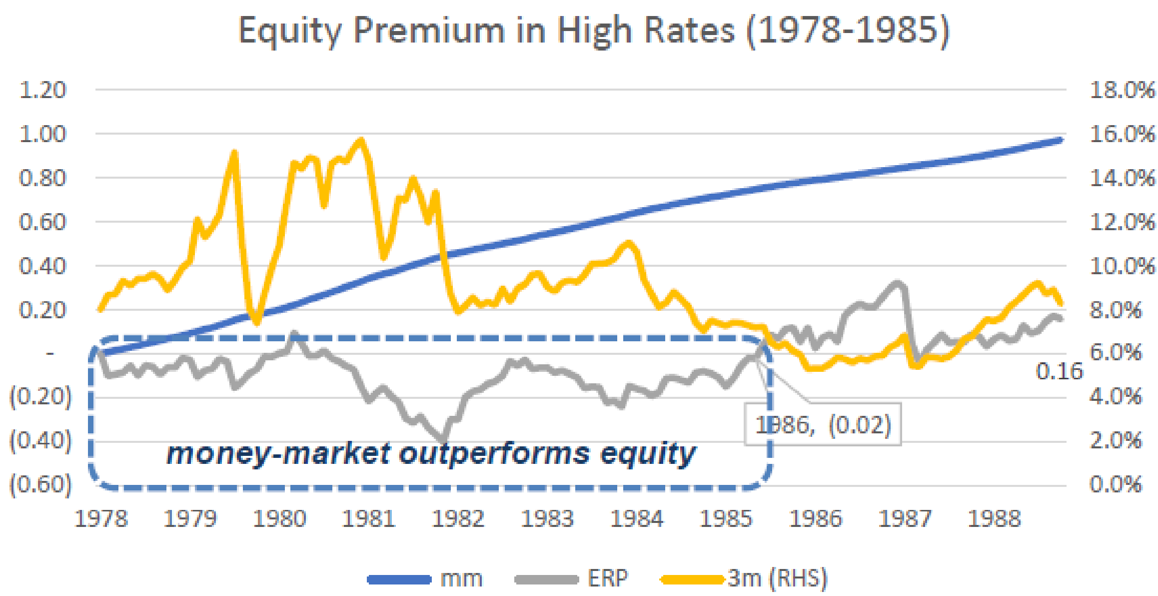


	Avg. ERP (RF)	*Avg. Return (RF)*	*Avg. Implied ERP*	*Avg. Implied Return*
Constant Return (AIRG)	*7.43% - RF*	*7.43%*	3.15%	7.43%
Constant ERP (GEMS)	*3.15%*	*RF + 3.15%*	3.15%	7.43%
Regression Line	*10.58% - 1.74 x RF*	*10.58% - .74 x RF*	3.13%	7.41%

- Above assumes: ***Equity Return* = *RF* + *ERP***, where RF is the return associated with risk-free rate, and ERP is the Equity Risk Premium or excess earned over RF.
- Analysis considers constant ERP, constant Return, and ERP as a linear function of risk-free rate.
- Rather than focusing on Equity Return by decile, this view of the data focuses on averaged realized ERP by decile.
- The chart suggests Constant Return assumption is better aligned with historical experience in tail deciles.

F. Historical Tails





- Cumulative Money Market return and ERP are presented on a continuous (log-return) basis, scaled on the left-hand side.
  - Example: wealth ratio (year 10) = exp(cumulative log return (year 10)).