The Probability of Default Model for Insurance Companies in the United States of America, Canada, and France

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Introduction

1. The following document describes the probability of default (PD) model and its implementation specifics for the insurance companies domiciled in the United States of America, Canada, and France. The PD model utilizes the forward-intensity corporate default prediction approach of Duan et al. (2012),[1] which is also the model underlying the NUS-CRI platform[2] for public firms globally. The document provides an overview of the preprocessing of data for the default predictors/attributes and credit event treatment. Also included are analysis of the model's performance and two use cases.

2. The key features of this PD model are as follows:

   a. Combines a reduced-form model (based on forward intensity construction) and a structural model that generates aggregate distance-to-default (DTD) measure for financial firms as one of the input covariates for the PD model.
   b. Accommodates two risks that a firm might encounter during the period of interest, namely the risk of default/bankruptcy and the risk of other types of corporate exits (such as the company dissolving due to voluntary management decisions). These two risks are modeled through two independent Poisson processes as detailed in Duan et al. (2012).[3]
   c. Employs multiple input covariates/predictors based on the raw data provided by AM Best, the data provider, in conjunction with other macro-financial factors from the NUS-CRI database (a complete list of the input covariates can be found in section 2).
   d. Enhances/complements the credit information provided by AM Best with NUS-CRI in-house resources to create a more complete/accurate dataset on defaults and other exits.
   e. Incorporates into the PD model a self-exciting feature, meaning that the realized default rate in a trailing period becomes an input variable for predicting default in the coming periods.

2. Data Preparation and Input Variables

3. This section documents the data preparation conducted by the NUS-CRI team on the classification of defaults and other exits as well as the input variables/predictors used in the PD model.

4. The raw data provided by AM Best included information pertaining to credit events, or events generally connected to a change in the business' operational nature, along with an annual time series of potential input variables[4] that can be used, after transformations, as potential predictors of defaults and other corporate exits.

[1] Duan, J. C., Sun, J., and Wang, T. (2012). "Multiperiod Corporate Default Prediction—A Forward Intensity Approach," *Journal of Econometrics*, 179, pages 191-209. DOI 10.1016/j.jeconom.2012.05.002.
[2] NUS-CRI Staff (2021). *NUS-CRI Technical Report Version: 2021 Update 1*. The Credit Research Initiative at the National University of Singapore (https://d.nuscri.org/static/pdf/Technical%20report_2021.pdf).
[3] Under the setting of two independent Poisson processes, joint occurrence of default/bankruptcy and other corporate exits has a zero probability, and the two types of risk in fact become competing.
[4] The raw data provided by AM Best that could have been used for potential predictors are as follows: Balance on combined technical account, Capital and surplus, Cash and deposits with credit institutions, Liquid assets, Long

2.1. Data for Corporate Events

5. To reasonably calibrate a PD model to the realized defaults in the data sample requires fairly clean data that accurately classifies company events as **Defaults** based on the standard market definition.

6. As companies can exit the market due to an event that is not a Default, and such an event critically influences survival of a firm, a similar data-cleaning exercise must take place to identify events that can be so classified as **Other Exits**.

7. If the company event does not constitute a Default or Other Exit, the company is said to have **Survived** the period of interest.

Table 1: Mapping the event type provided by AM Best to three classes

Event Type	Default/Other Exit/Survival
Domiciliary Change	Survival
Ownership	Other Exit
Merged	Other Exit
Name Change	Survival
In Liquidation	Default
Sold as Shell	Other Exit
Ceased Operation	Other Exit
Dissolved*	Default/Other Exit
Portfolio Transfer	Survival
No Longer Exists	Other Exit
In Runoff	Survival
Surrendered License	Survival
Liquidated	Default
No Longer Filing	Survival
Other	Other Exit
Suspended	Other Exit
License Revoked	Other Exit
Domiciliary Change	Survival

* For dissolved cases, due to some firms dissolving after a default rather than dissolving due to other business or management reasons, investigations carried out by the NUS-CRI team have classified them accordingly into Defaults or Other Exits.

8. There are special considerations for companies that have been liquidated or are in liquidation. Because a company generally enters a rehabilitation phase prior to liquidation, companies that are in the process of being rehabilitated have already been facing financial difficulty and have typically missed payment on their obligations. As such, the effective date of default for those companies that ultimately end up being liquidated is taken as the date that the company enters rehabilitation.

9. Furthermore, companies that have been dissolved may have been dissolved due to either a default or voluntary management decisions. For this data set, there are close to 550 dissolved

term borrowings, Short term borrowings, Profit (Loss) before tax, Total liabilities and surplus, and Total assets. The data provided for each firm follows a time series on an annual frequency from 2007 to 2021.

cases that the NUS-CRI team had manually checked to classify them as either Default or Other Exits, respectively.

10. Further investigation has been conducted for those credit events with incomplete effective dates. The raw data has close to 50 such events, either only having the year of event, or the month and the year of event. The NUS-CRI team has conducted further manual checks to gather more information on these companies' event dates. If a complete date is not found, assumptions are made that the credit event took place in the middle of the effective year or month provided in the raw data.

11. According to the above classification, Table 2 shows the summary statistics on the cleaned-up data sample. A more detailed breakdown of the corporate events time series for each country is displayed in Appendix A:

Table 2: Summary statistics for the clean-up data sample

# of insurance companies	8,039
Time period	02/2008 – 12/2021
# of defaults	168
# of other exits	1,355
# of firm-month observations	976,108

2.2. Input Variables

12. The input variables used in the PD model include common macro-financial variables, firm-specific variables, and self-exciting measures (see Table 3).

13. Following the NUS-CRI PD model, common macro-financial variables include interest rates,[5] stock index return,[6] and financial aggregate DTD,[7] which are retrieved from the NUS-CRI database.

14. The firm-specific variables are constructed in terms of four different firm characteristics.

    a. Liquidity = log(Liquid assets / Total assets)
    b. Profitability = Profit/(loss) before tax / Total assets
    c. Debt Position = Long term borrowings / TL, where TL (Total liabilities) = Total liabilities & surplus - Capital & surplus and the negative values of Long term borrowings would be treated as missing (ineffective data)
    d. Relative Size = log(Total assets / MTA), where MTA is median of Total assets in each month

[5] *interest rate*: a representative 3-month short-term interest rate. The interest rates used for U.S., Canada, and France are U.S. Generic Govt 3 Month Yield, Canada Treasury Bill 3 Month, and Germany 3 Month Bubill, respectively.
[6] *stock index return*: the trailing one-year simple return on a major stock index of the economy. The stock indices used for U.S., Canada, and France are S&P 500 Index, S&PTSX Composite Index, and CAC 40 Index, respectively.
[7] *financial aggregate DTD*: median DTD of financial firms in each economy inclusive of those foreign financial firms whose primary stock exchange is in this economy, where DTD is a measurement of volatility-adjusted leverage based on Merton-type model and the details can be found in the NUS-CRI Technical Report (Version 2021 update 1).

15. The PD model has incorporated a self-exciting feature with two measures on the target portfolio, i.e., insurance companies in the U.S., Canada, and France. These two measures are (1) the 12-month moving average of 1-month realized default rates and (2) the current 1-month realized default rate minus the 12-month moving average (which we denote as *trend*).

16. In addition to the above-mentioned common variables and firm-specific variables, a dummy variable is added to indicate whether a firm is in North America (NA), i.e., it attains a value of 1 for US and Canadian firms and 0 for French firms. After constructing the list of input variables (the summary is shown in Table 6), the following data treatment has been performed:

    a. Fitting data to monthly frequency: For the annual accounting data, we assume it is available in February in the following year and monthly firm-specific variables are constructed using the annual accounting information. For daily data such as interest rates and stock index values, the last day of the month for which there is valid data is used.
    b. Treatment for outliers: Winsorization is performed to eliminate outliers by applying a floor and a cap on each of the firm-specific attributes except for the dummy variables. The historical 0.1 percentile and 99.9 percentile for the whole sample are recorded and any values that exceed these levels are set to these boundary values.
    c. Treatment for missing values: There is a high proportion (≈80%) of missing values for Long term borrowings, and the missing values are assigned zero. The potential bias arising from this assignment is handled by introducing a corresponding dummy variable with a value of 1 to indicate non-missing cases in order to offset the overall non-zero value effect.

17. Appendix A provides summary statistics of the firm-specific variables.

Table 3: Summary of input variables/predictors used in the PD model*

Input Variables		
1	Common macro-financial variables	Three-month Interest Rates in Canada
2		Three-month Interest Rates in France
3		Three-month Interest Rates in U.S.
4		Economy-specific Stock Index Return
5		Economy-specific Aggregate DTD of Financial Firms
6	Self-exciting variables	12-month Moving Average of 1-month Realized Default Rates (MA)
7		Trend
8	Firm-specific variables	Liquidity
9		Profitability
10		Debt Position
11		Relative Size
12	Dummy variables	Dummy for North America (NA)
13		Dummy for Debt Position

* There are 13 input variables in total, including 5 macro-financial indicators, 2 self-exciting variables, 4 firm-specific variables and 2 dummies. Considering that interest rates would have different impacts in different economies, each country's coefficient is individually estimated.

3. The Model

18. The PD model follows the forward intensity corporate default prediction approach introduced in Duan et al. (2012) with the reference given in Footnote 1, which underlies the NUS-CRI corporate default prediction system (see Footnote 2). The forward intensity approach is a reduced form model in which the intensities driving the PD term structure are computed as different functions of various input variables already introduced in Section 2.2. This forward intensity model is governed by two independent Poisson processes with time-varying parameters (one for default and the other for other exit), operating on forward time instead of spot time. To be more specific, a firm's default / other exit is signaled by a jump in the Poisson process and the probability of such a jump is determined by the intensity of the Poisson process. The forward intensity model draws an explicit dependence of the intensities at time periods in the future (i.e., forward intensities) to the values of the input variables at the time of prediction. This enables the model to produce forward-looking PD-term structures for firms based on dynamic learning from the macro-financial and firm-specific data. In the current implementation, PDs are forecast from a horizon of one month up to a horizon of 10 years.

19. In this PD model for insurance companies, we tweak the forward intensity approach of Duan et al. (2012) by introducing into the PD model a self-exciting feature, meaning that the realized default rate in a trailing period becomes an input variable for predicting default in the coming periods. This self-exciting feature turns out to significantly enhance the model's performance. It should be noted that the self-exciting feature destroys the original property of doubly stochastic Poisson processes where stochastic intensities do not face a feedback loop from subsequent Poisson jumps. Hence, the model here in essence creates a forward intensity version of Hawkes processes.

20. When the forward intensities are in place, the conditional forward probabilities can be easily calculated. For each forward starting time τ, firm i's forward PD at time t for a future time period $(t+\tau, t+\tau+1)$, denoted by $p_{i,t}\,(\tau)$, is constructed on a forward intensity function, whose inputs $\boldsymbol{X_{i,t}}$ include the state of the economy (the macro-financial factors), the vulnerability of individual firms (firm-specific attributes) and the self-exciting component:

$$p_{i,t}\,(\tau) \;= Prob_t(Y^{(i)}_{t+\tau,t+\tau+1} = 1) \;=\; f(\boldsymbol{X_{i,t}}; \tau, \boldsymbol{\theta})$$

where $Y^{(i)}_{t+\tau,t+\tau+1} = 1$ represents that the firm i defaults in the future time period $(t+\tau, t+\tau+1)$. Similarly, $q_{i,t}\,(\tau)$, which is firm i's forward POE at time t for a future time period $(t+\tau, t+\tau+1)$ can be written as the function below:

$$q_{i,t}\,(\tau) = Prob_t(Y^{(i)}_{t+\tau,t+\tau+1} = 2) \;=\; g(\boldsymbol{X_{i,t}}; \tau, \overline{\boldsymbol{\theta}})$$

where $Y^{(i)}_{t+\tau,t+\tau+1} = 2$ represents that the firm i has a non-default exit in the future time period $(t+\tau, t+\tau+1)$. The parameters $(\boldsymbol{\theta}, \overline{\boldsymbol{\theta}})$ can be calibrated by maximizing the log-likelihood of the data sample of all companies as follows:

$$L(\boldsymbol{\theta}, \overline{\boldsymbol{\theta}}; \tau) = \sum_{t\in T}\sum_{i=1}^{N}(1_{\{Y^{(i)}_{t+\tau,t+\tau+1}=1\}}\ln\left[f\left(\boldsymbol{X_{i,t}}; \tau, \boldsymbol{\theta}\right)\right] + 1_{\{Y^{(i)}_{t+\tau,t+\tau+1}=2\}}\ln\left[g(\boldsymbol{X_{i,t}}; \tau, \overline{\boldsymbol{\theta}})\right] + 1_{\{Y^{(i)}_{t+\tau,t+\tau+1}=0\}}\ln\left[1 - f\left(\boldsymbol{X_{i,t}}; \tau, \boldsymbol{\theta}\right) - g(\boldsymbol{X_{i,t}}; \tau, \overline{\boldsymbol{\theta}})\right])$$

where $Y^{(i)}_{t+\tau,t+\tau+1} = 0$ represents that the firm i survives the future time period $(t+\tau, t+\tau+1)$, T is the sample time period and N denotes the total number of insurance companies.

21. Optimization must factor in the high dimensionality of the parameters (i.e., 13 covariates[8] for 120 monthly prediction horizons). We deploy a Nielson-Siegel term structure function on this input variable/predictor and rely on sequential Monte Carlo optimization for the model's calibration. Details of the procedure can be found in the NUS-CRI Technical Report (see Footnote 2). Parameter estimates corresponding to an input variable/predictor for the entire horizon up to 10 years for default and non-default exits can then be deduced directly from the NS function.

22. Some input variables have an unambiguous effect on a firm's PD. For example, increments of liquidity and profitability should indicate that a firm is becoming more creditworthy, leading to a decreasing PD. As a result, the default parameters associated with liquidity and profitability at all 120 horizons are constrained to be negative. A negative default parameter at a forward starting time means that if the value of that variable increases, the forward default intensity decreases and the corresponding conditional forward PD decreases.

4. Performance Highlights

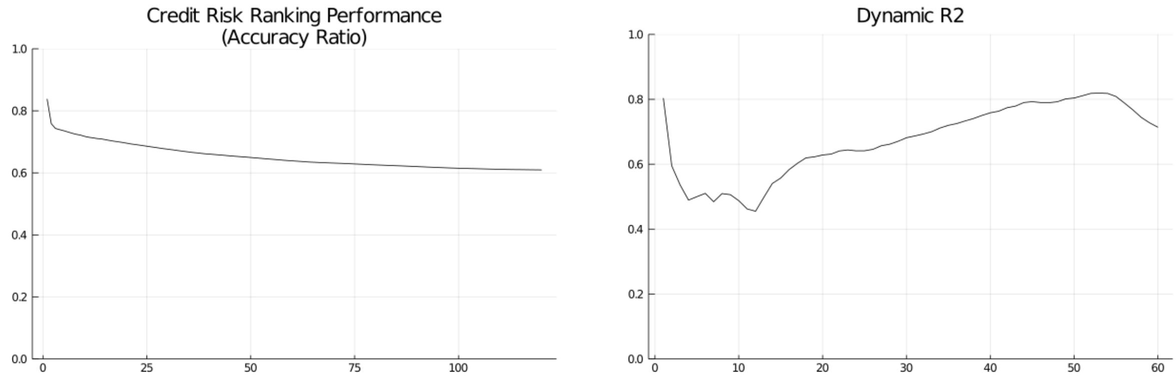
4.1 Credit risk ranking performance (Accuracy Ratio)

23. Figure 1 (left panel) displays the credit risk ranking performance using accuracy ratios,[9] abbreviated as AR hereinafter, from 1-month to 10-year horizon. As expected, the AR for this model is higher in the short term (83.80% for 1-month prediction horizon), and naturally reduces in the longer term (61.27% for 10-year prediction horizon).

[8] There are 14 parameters for the 13 input variables and an intercept for each prediction horizon.

[9] AR is the ratio of 'ar' over 'ap,' where the former is the area between the Cumulative Accuracy Profile (CAP) of the rating model and the CAP of the random (totally uninformed) model, and the latter is the area between the CAP of the perfect model and the CAP of the random model. The CAP is obtained by first ordering the PDs in a descending order. Then, for a given fraction x of the total number of firms, the CAP curve indicates the fraction of the defaulted firms whose PDs are greater than or equal to the minimum PD up to fraction x, where fraction x will be varied from 0% to 100%. To appreciate the magnitude of an AR, we note that the AR for a totally uninformed model is 0. One may occasionally encounter a different performance measure known as ROC, which assigns a totally uninformed model an ROC of 50% leading to an impression of better performance.

Figure 1: Credit risk ranking performance and dynamic R^2



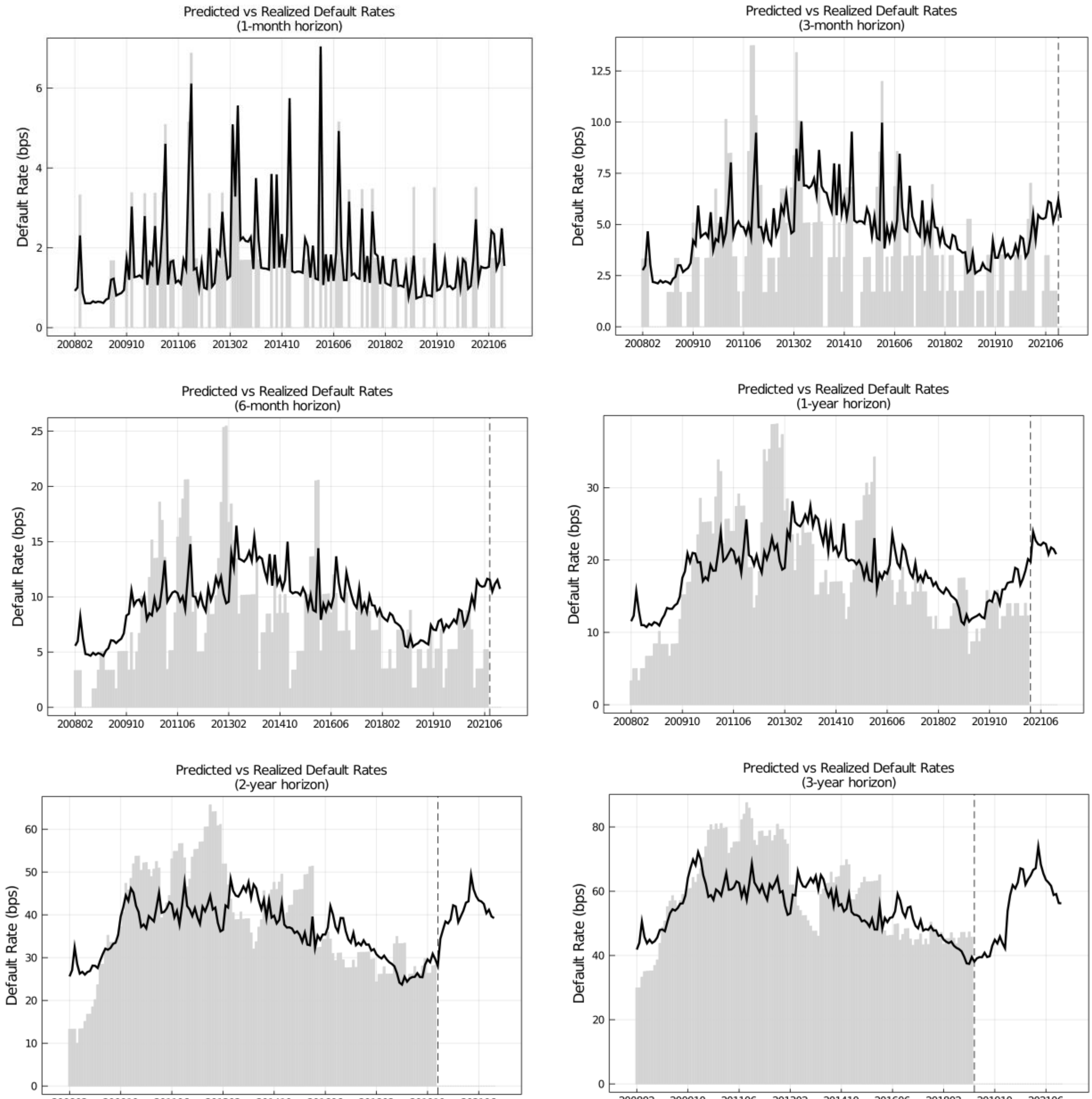
4.2 Predicted vs. realized default rates and dynamic R^2

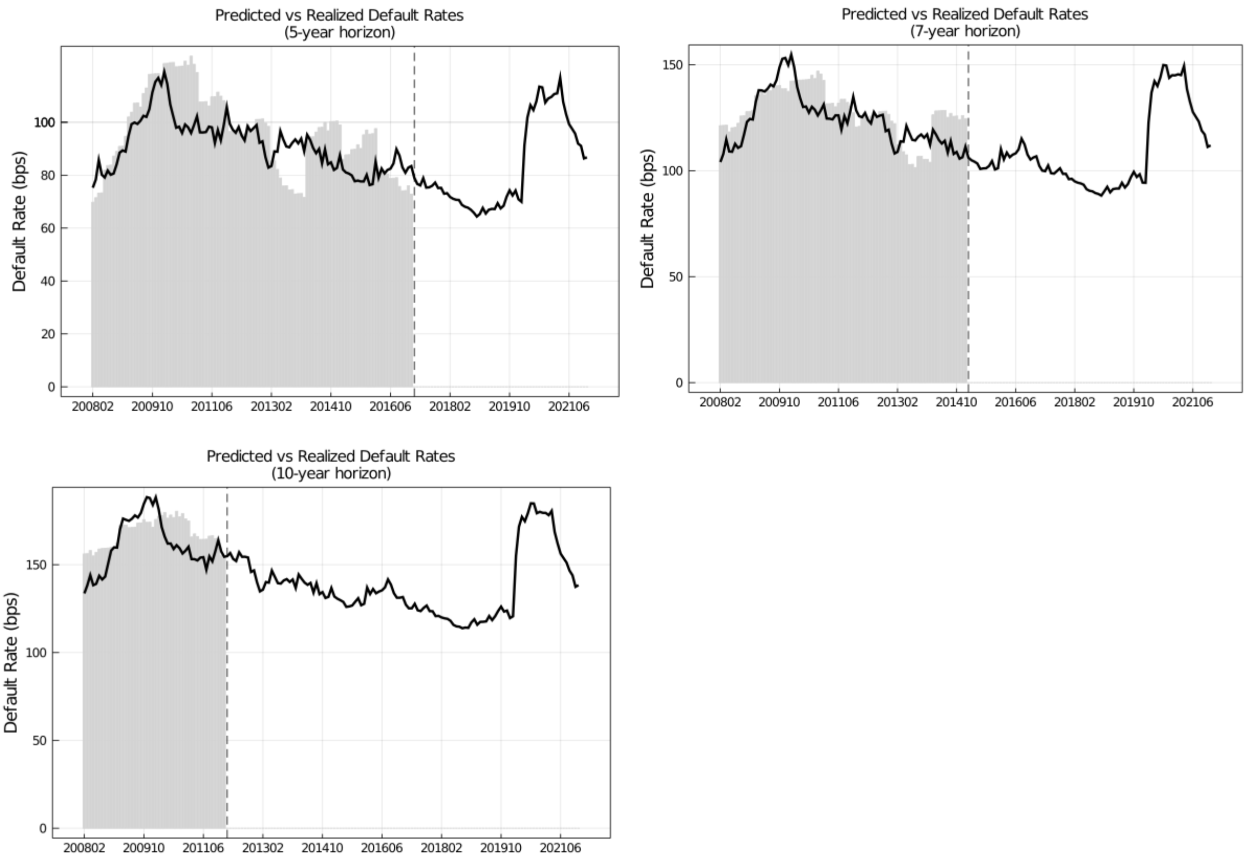
24. For further analysis, a time series performance measure can quickly summarize the performance in an R^2 style on the portfolio in the time dimension, referred to as the dynamic R^2.[10] In this case, the dynamic R^2 is calculated up to the 5-year horizon because beyond 5 years, there are insufficient data points to produce meaningful results. Figure 1 (right panel) shows the dynamic R^2 for each horizon from 1 month to 5 years, which varies from 47% to 78.62%, respectively.

25. Figure 2 shows the predicted vs realized default rates for some usual prediction horizons of interest, which are 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, and 10-year, respectively. For ease of comparison in this figure, we have advanced the predicted default rates by their respective prediction horizon to align them with their respective intended period, where the realized default rates are tallied. Naturally, the realized default plot ends earlier because beyond a certain point of time, the corresponding realized defaults are not yet available for tallying. In the figures, the black vertical dashed line shows the boundary, beyond which there is no data to calculate realized default rates.

[10] This dynamic R^2-type measure is defined to be 1 minus the ratio of two items where the first item is the sum of squares of prediction errors of the PD model (realized default rate minus its corresponding predicted probability), and the second item is the sum of squares of prediction errors of the naive prediction (where predicted probability is the trailing realized default rates of corresponding horizons). Using default rates instead of default numbers in this measurement is for standardization because the number of borrowers varies over the sample period.

Figure 2: Predicted vs realized default rates*





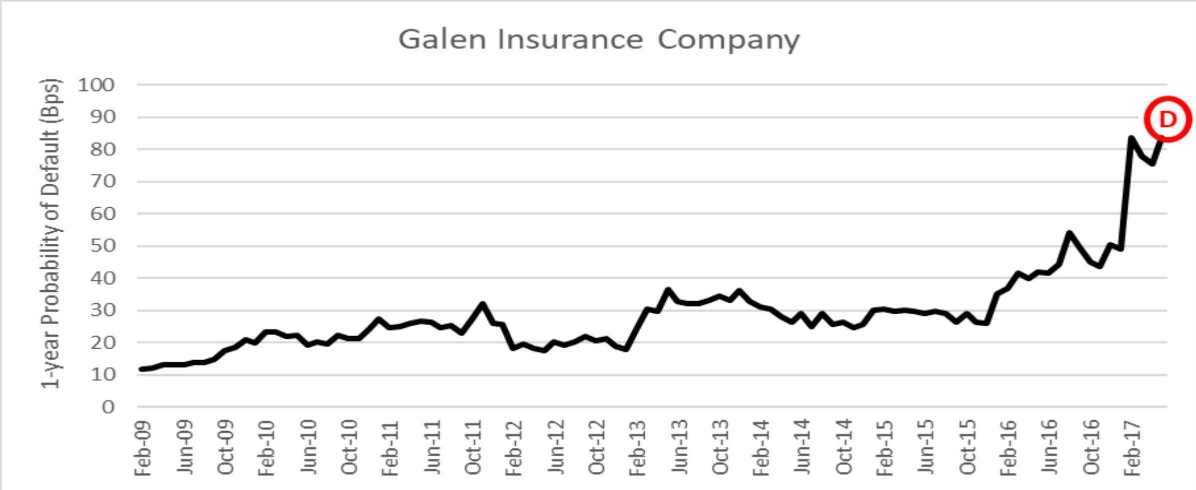
* 1-month to 10-year horizons, black solid line: the predicted default rates in bps, gray bar: realized default rates in bps

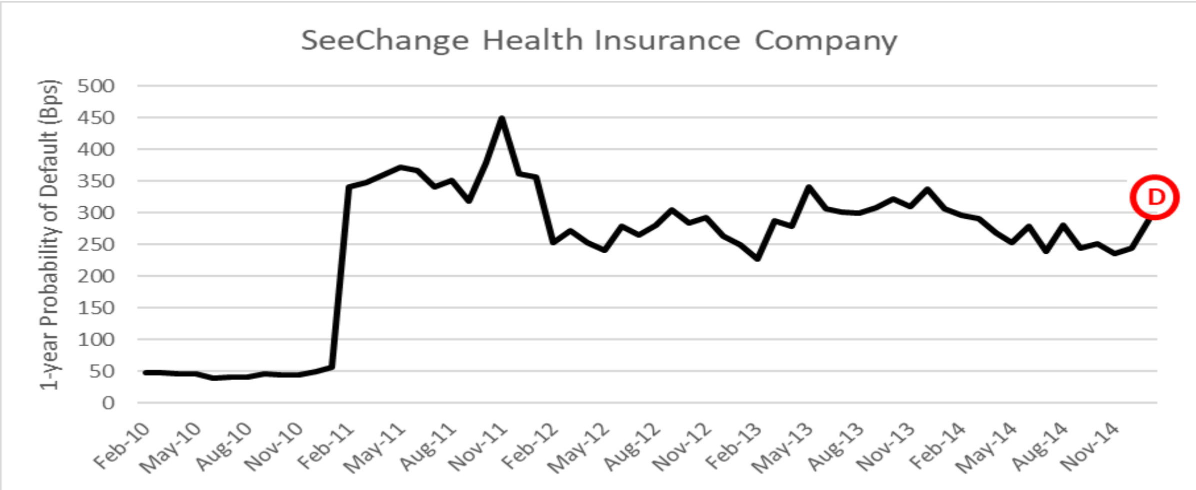
5. Use of the PD Model

26. It is informative to see how a model can be used through a case study by visually checking individual companies' PDs on, say, a 12-month horizon. Specifically looking at the time series trend prior to its default can help gauge whether the PD model has default prediction capabilities.

27. Figure 4 displays results on two companies that have entered liquidation or rehabilitation in the sample. As we can see, both plots are able to capture the increase in each company's credit risk prior to its default.

Figure 4: PD time series of two insurance firms prior to their defaults





6. Appendix

A. Summary Statistics on default distribution and parameter estimates

Table 4: Number of defaults and other exits in The United States

Country: The United States					
Year	# of companies	Defaults		Other Exits	
		#	%	#	%
2008	5404	2	0.04	99	1.83
2009	5354	7	0.13	72	1.34
2010	5361	15	0.28	86	1.60
2011	5243	17	0.32	63	1.20
2012	5345	9	0.17	78	1.46
2013	5331	24	0.45	113	2.12
2014	5288	17	0.32	84	1.59
2015	5262	11	0.21	105	2.00
2016	5229	16	0.31	118	2.26
2017	5160	14	0.27	84	1.63
2018	5144	9	0.17	88	1.71
2019	5146	9	0.17	93	1.81
2020	5173	11	0.21	64	1.24
2021	5218	5	0.10	64	1.23

Table 5: Number of defaults and other exits in Canada

Country: Canada					
Year	# of companies	Defaults		Other Exits	
		#	%	#	%
2008	346	0	0	10	2.89
2009	372	0	0	13	3.49
2010	380	0	0	5	1.32
2011	370	0	0	2	0.54
2012	380	1	0.26	8	2.11
2013	384	0	0	13	3.39
2014	370	0	0	7	1.89
2015	363	0	0	11	3.03
2016	350	0	0	12	3.43
2017	338	0	0	11	3.25
2018	327	0	0	3	0.92
2019	323	0	0	7	2.17
2020	313	0	0	1	0.32
2021	304	0	0	2	0.66

Table 6: Number of defaults and other exits in France

Country: France					
Year	# of companies	Defaults		Other Exits	
		#	%	#	%
2008	283	0	0	5	1.77
2009	274	0	0	4	1.46
2010	268	0	0	6	2.24
2011	269	0	0	6	2.23
2012	272	1	0.37	5	1.84
2013	288	0	0	9	3.13
2014	313	0	0	7	2.24
2015	331	0	0	10	3.02
2016	339	0	0	8	2.36
2017	341	0	0	10	2.93
2018	326	0	0	7	2.15
2019	320	0	0	13	4.06
2020	288	0	0	5	1.74
2021	264	0	0	0	0.00

Table 7: Summary statistics of firm-specific financial variables

Liquidity

Country	Min	25%	Median	75%	Max	Mean	StdDev	Observations
Canada	-14.3208	-0.60027	-0.34309	-0.12537	0.101383	-0.54404	0.934069	57799
France	-14.3208	-0.74391	-0.40143	-0.1654	0	-0.85235	1.661598	49108
US	-14.3208	-0.40742	-0.20402	-0.08704	0.361204	-0.36552	0.677176	870401

Profitability

Country	Min	25%	Median	75%	Max	Mean	StdDev	Observations
Canada	-1.4431	0.006056	0.023327	0.050553	9.480399	0.035169	0.187325	57966
France	-2.26807	0.002999	0.009489	0.032987	0.919499	0.022382	0.080754	49274
US	-4560.83	0.001659	0.023799	0.057514	62.53334	0.004013	10.94224	869344

Relative Size

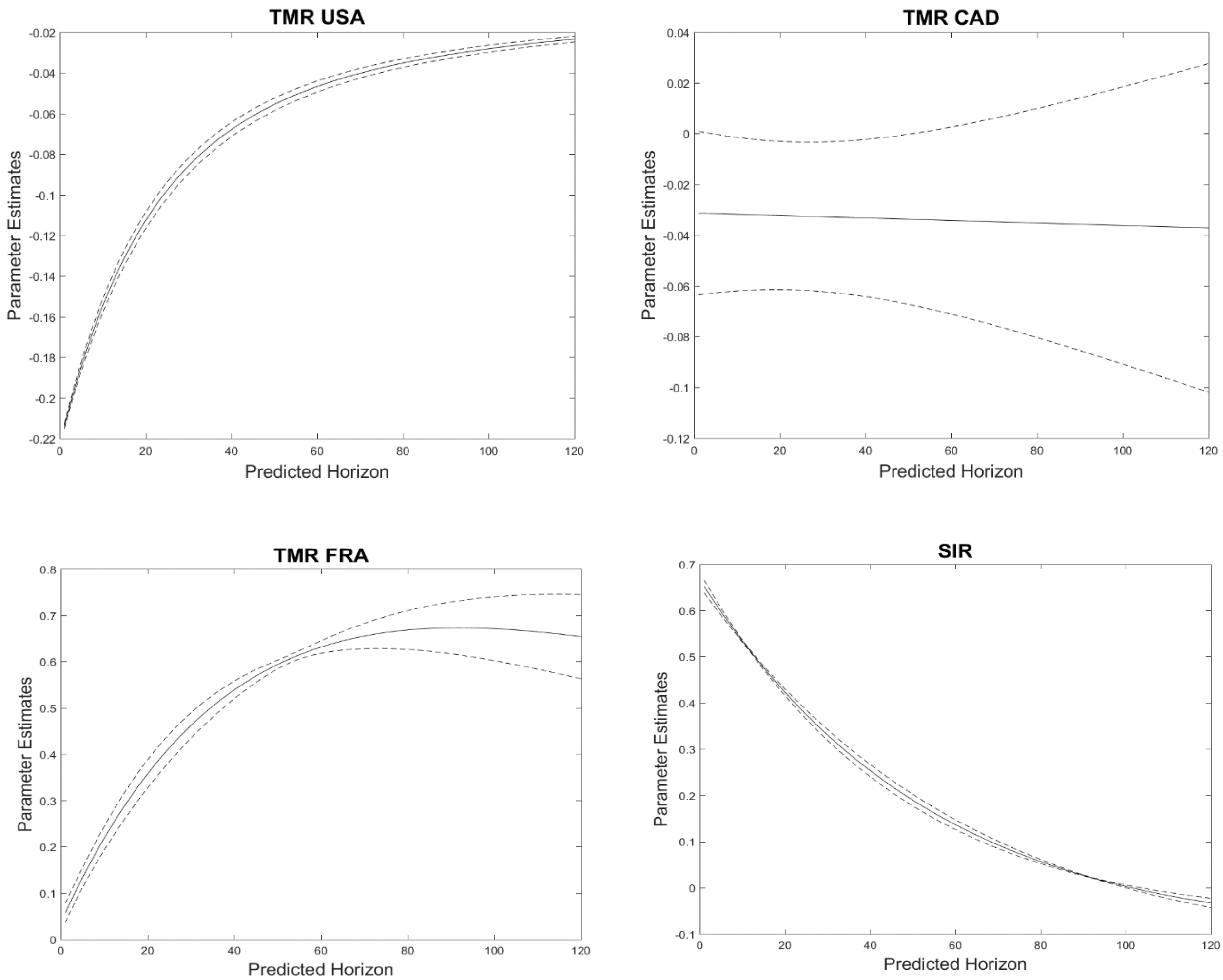
Country	Min	25%	Median	75%	Max	Mean	StdDev	Observations
Canada	-412.124	-1.29279	0.417088	2.021895	8.250739	-1.37721	28.37121	58241
France	-412.025	0.479474	2.314436	4.016916	9.112241	2.150625	6.930376	49369
US	-412.091	-1.90188	-0.15894	1.678616	9.096684	-0.04419	4.838823	870555

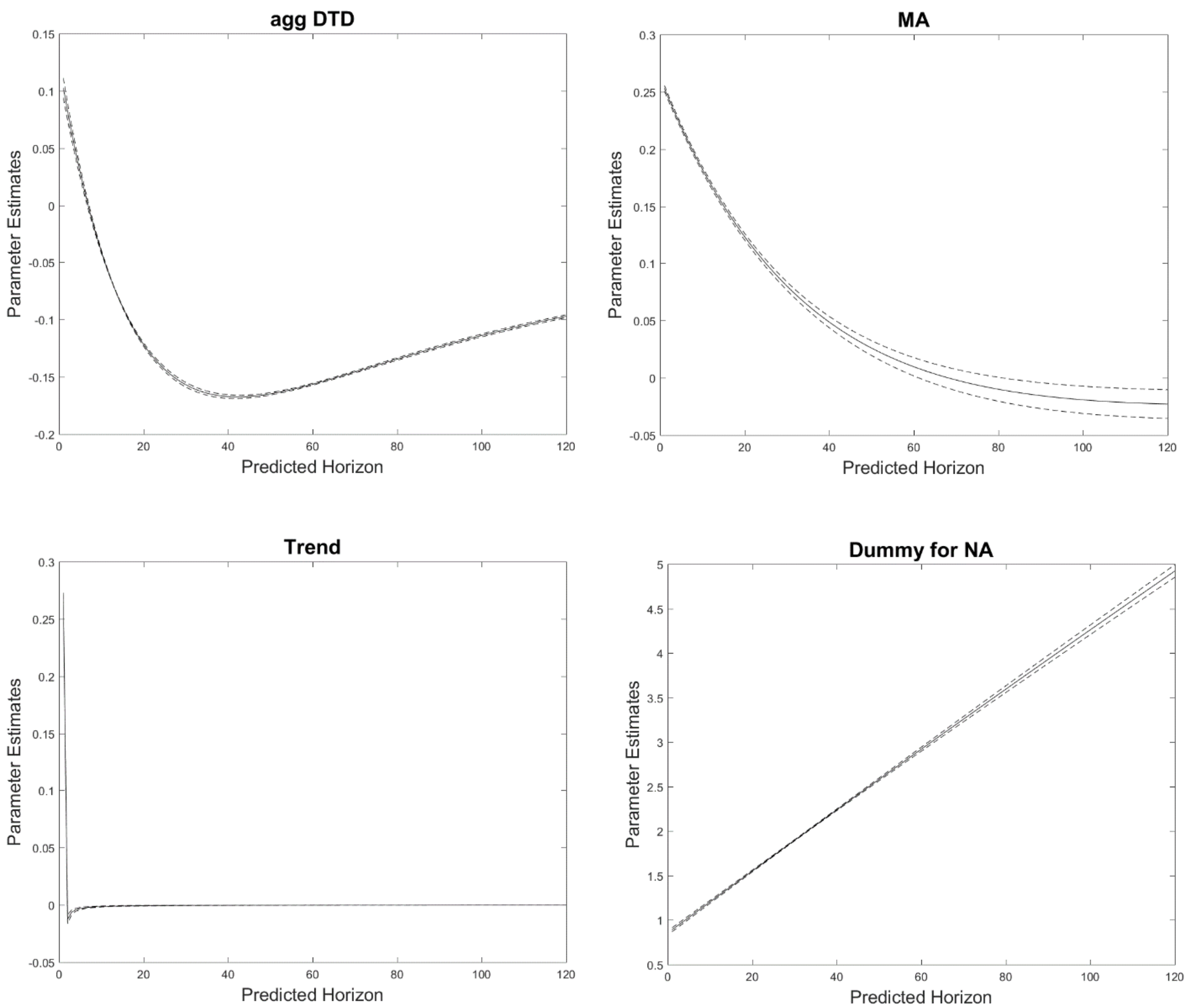
Debt Position								
Country	Min	25%	Median	75%	Max	Mean	StdDev	Observations
Canada	4.11E-06	0.004928	0.016039	0.03583	33.37784	0.194056	1.897677	4108
France	1.40E-08	0.000147	0.00091	0.016358	0.997391	0.026252	0.087409	22174
US	6.95E-09	0.003008	0.026629	0.08682	71.31309	0.09681	0.901492	99248

B. Parameter Estimates

28. Figure 5 plots the default parameters across all horizons for common macro-variables. These include the Three-month Interest Rates (TMR) for each country, Stock Index Return (SIR), Aggregate Distance to Default (Agg DTD), 12-month Moving Average of 1-month Realized Default Rate (MA) and its Trend, and NA Dummy. The corresponding brief explanation for each parameter is available in Section B.1 to B.5.

Figure 5: Default parameters across all horizons for common variables*

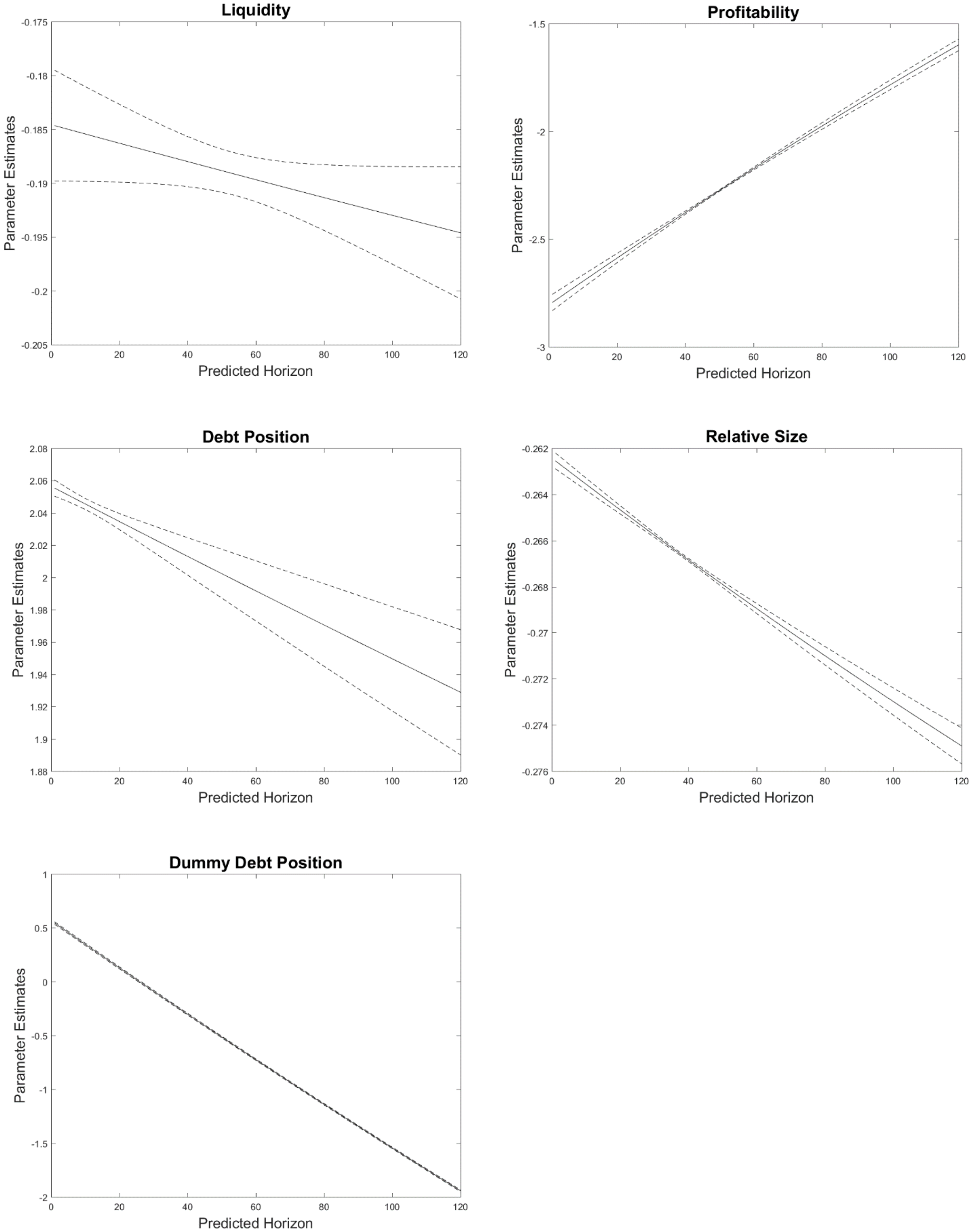




* Solid lines are the parameter estimates and dashed lines are the 90% confidence bands. Horizontal axis is the horizon in months.

29. Figure 6 provides the model parameters for all the firm-specific variables constructed using the data provided by AM Best. Their corresponding explanations are presented in Section B.6 and B.7.

Figure 6: Default parameters across all horizons for firm-specific variables*



* Solid lines are the parameter estimates and dashed lines are the 90% confidence bands. Horizontal axis is the horizon in months.

B.1 Three-month Interest Rates (TMR)

30. There are three parameters in total to describe the effect of three-month interest rate on PD, one for each of the three countries in the data sample (the U.S., Canada, and France). They are modeled as three separate input variables because their differences in magnitude call for different response coefficients. The findings suggest that the TMR for the U.S. has a negative effect on PD prediction as the parameters are generally negative. The parameter of TMR for France shows an opposite impact on PD. The impact of TMR for Canada on PD is negative but marginally insignificant.

B.2 Stock Index Return (SIR)

31. Model parameters are positive for the impact of SIR on PD. On the surface, it suggests that a higher trailing stock market return raises credit risk in general. However, this might simply be due to its role in offsetting an over-valuation effect (i.e., equity value run-up) that has caused inflated distance-to-default.

B.3 Aggregated Distance to Default for financial firms (Agg DTD)

32. Model parameters are generally negative, except for short horizons, for the impact of Agg DTD on PD. This is in line with the intuition that in a credit environment where financial firms have a higher distance-to-default on aggregate, financing becomes more readily available and credit conditions are generally loose, leading to lower credit risks in the short run but higher risks in the long run.

B.4 Moving Average of 1-month Realized Default Rate (MA) and corresponding trend

33. The newly introduced self-exciting feature is helpful in improving default prediction. This feature is captured by two measurements—the level and trend in the trailing realized one-month default rates. As seen in Figure 5, the relationship between MA and PD is positive in the short run, while getting smaller in the longer run. Trend has no effect except for a very short horizon.

B.5 Dummy for North America (NA)

34. North America (inclusive of U.S.- and Canada-domiciled insurers) might face different shocks than France would. A dummy variable that accounts for this geographical difference and difference in economic shocks is thus introduced. The parameters for the NA dummy are positive, signaling that firms that are domiciled in North America face a higher credit risk than those domiciled in France, ceteris paribus, and the difference diminishes when the horizon becomes longer. This reflects a data feature of this sample in which the default rate in the U.S. is far greater than that in France.

B.6 Liquidity, Profitability, and Relative Size

35. In line with economic intuitions, our model parameters show a negative relationship between liquidity, profitability, and relative size with PD. As firms increase their liquidity position, their credit worthiness should improve as they have more liquid assets to meet their obligations. Furthermore, should a firm’s profitability increase, its credit risk will naturally decrease.

Similarly, as the relative size of a firm compared to the rest of its peers increases, the firm is deemed to be safer as it may be considered to have more diversified revenue sources, a better access to financing, and/or a greater chance of receiving governmental assistance.

B.7. Debt Position

36. As this variable has a dummy to treat its missing value, the analysis pertaining to the impact of the debt position should reflect differential impacts that missing and non-missing cases might have. The debt position parameter in itself has a positive relationship with PD in line with the intuition. For shorter horizons, positive model parameters on the dummy for companies with a debt position value, suggesting that the company's PD increases compared to a company that has missing debt position value. For longer horizons, the relationship reverses.

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