AMERICAN ACADEMY *of* ACTUARIES

CONSTRUCTION AND USE OF PRE-PACKAGED SCENARIOS TO SUPPORT THE DETERMINATION OF REGULATORY RISK-BASED CAPITAL REQUIREMENTS FOR VARIABLE ANNUITIES AND SIMILAR PRODUCTS

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The following report is a follow up to a proposal from March 2002 and was prepared by the Life Capital Adequacy Subcommittee's C-3 Phase 2 Work Group* (chaired by Bob Brown). The work group is made up of several members of the subcommittee as well as Mike Akers, Frederick Andersen, Robert Berendsen, Tom Campbell, Andrew Eastman, Jack Gies, Geoffrey Hancock, Jeff Krygiel, Jim Lamson, Jeffrey Leitz, Craig Morrow, John O'Sullivan, Link Richardson, Jim Reiskytl, Dave Sandberg, Van Villaruz, and Albert Zlogar. The work group would also like to thank Philip Barlow, Jan Brown, Bill Carmello, Michael Cebula, Allen Elstein, Mark Evans, Dennis Lauzon, and Mark Tenney for their helpful suggestions and feedback.

(*It should also be noted, that since this project has been on-going for several years, many other individuals contributed to the work that paved the way for this report.)

Background

Over the past two years, the AAA Life Capital Adequacy Subcommittee (LCAS) has issued several versions of a report entitled "*Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products*"[1] that recommends implementing "C3 Phase II RBC" to address both the interest rate and equity risk associated with variable annuity products with guaranteed benefits.

The LCAS recommendation indicates that the American Academy of Actuaries will provide 10,000 "pre-packaged" scenarios for the common asset classes typically needed in the stochastic cashflow projections of variable annuities. This report documents the construction (i.e., models and parameters) of the "pre-packaged" scenarios and also provides guidance on their use. Two utilities are included in the package of materials: (1) a tool that allows companies to select (pick) a subset of representative scenarios from the full sample of 10,000; and (2) an enhanced version of the C-3 Phase I RBC interest rate generator.

This package is an enhancement to earlier versions of the pre-packaged scenarios in that this latter utility affords the company the flexibility to input the starting U.S. Treasury yield curve and re-generate scenarios (interest rates and bond index returns) consistent with that initial term structure. However, only the interest rate scenarios (and consequently, index returns for Fixed Income and Balanced funds) have changed to reflect a different starting yield curve. All other scenario components (e.g., equity index returns, random samples, etc.) remain the same as for the March 2005 pre-packaged scenarios.

While the LCAS proposal encourages companies to develop their own stochastic models for scenario testing, companies may choose to use the pre-packaged scenarios as an alternative. Also, the "Alternative Method" factors and formulas for Guaranteed Minimum Death Benefit (GMDB) risks have been developed from stochastic testing using these scenarios[2]. More information on the scenario files is provided later in this document under "Pre-packaged Scenario Files."

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[1] Variations on this title include "*Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Products with Guarantees (Excluding Index Guarantees)*"

[2] The factor-based Alternate Methodology for GMDBs used the March 2005 version of the pre-packaged scenarios.

Scenario Files

Scenario files are provided for the economic variables (asset classes) shown in Table 1A.

Table 1A: Scenario Files

Asset Class	Short Name	Scenario File
3-month U.S. Treasury yields	UST_3m	UST_3m.csv
6-month U.S. Treasury yields	UST_6m	UST_6m.csv
1-year U.S. Treasury yields	UST_1y	UST_1y.csv
2-year U.S. Treasury yields	UST_2y	UST_2y.csv
3-year U.S. Treasury yields	UST_3y	UST_3y.csv
5-year U.S. Treasury yields	UST_5y	UST_5y.csv
7-year U.S. Treasury yields	UST_7y	UST_7y.csv
10-year U.S. Treasury yields	UST_10y	UST_10y.csv
20-year U.S. Treasury yields	UST_20y	UST_20y.csv
30-year U.S. Treasury yields	UST_30y	UST_30y.csv
Money Market / Short-Term	MONEY	MONEY.csv
U.S. Intermediate Term Government Bonds	U.S. ITGVT	ITGVT.csv
U.S. Long Term Corporate Bonds	U.S. LTCORP	LTCORP.csv
Diversified Fixed Income	FIXED	FIXED.csv
Diversified Balanced Allocation	BALANCED	BALANCED.csv
Diversified Large Capitalized U.S. Equity	US	US.csv
Diversified International Equity	INTL	INTL.csv
Intermediate Risk Equity	SMALL	SMALL.csv
Aggressive or Specialized Equity	AGGR	AGGR.csv

Random Numbers

The correlated random samples for the stochastic innovations (i.e., "noise" terms) are provided in the files shown in Table 1B. All samples are from the multivariate standard normal distribution (i.e., each marginal distribution is normal with zero mean and unit variance). The stochastic processes (models) are explained later in this document.

Table 1B: Random Number Files

Asset Class	Model Factor	Random Number File
Money Market / Short-Term	Nominal Return	rand_ret_MONEY.csv
U.S. Intermediate Term Government Bonds	Nominal Return	rand_ret_ITGVT.csv
U.S. Long Term Corporate Bonds	Nominal Return	rand_ret_LTCORP.csv
Diversified Large Capitalized U.S. Equity	Log Return	rand_logret_US.csv
Diversified Large Capitalized U.S. Equity	Volatility	rand_vol_US.csv
Diversified International Equity	Log Return	rand_logret_INTL.csv
Diversified International Equity	Volatility	rand_vol_INTL.csv
Intermediate Risk Equity	Log Return	rand_logret_SMALL.csv
Intermediate Risk Equity	Volatility	rand_vol_SMALL.csv
Aggressive or Specialized Equity	Log Return	rand_logret_AGGR.csv
Aggressive or Specialized Equity	Volatility	rand_vol_AGGR.csv

Equity Market Volatility

The "realized equity market volatilities" (annualized, by month) generated by the stochastic log volatility (SLV) model (see later under "Equity Market Returns") are provided in the files shown in Table 1C.

Table 1C: Random Number Files

Asset Class	Model Factor	Volatility File
Diversified Large Capitalized U.S. Equity	Realized Volatility	vol_US.csv
Diversified International Equity	Realized Volatility	vol_INTL.csv
Intermediate Risk Equity	Realized Volatility	vol_SMALL.csv
Aggressive or Specialized Equity	Realized Volatility	vol_AGGR.csv

Market Data

Table 2 indicates the composition or reference market for each asset class. Parameters are determined by fitting the models to data over the specified historic periods. Historic data were obtained from Ibbotson Associates.

Table 2: Asset Classes for Scenario Modeling

Asset Class	Market Proxies	Historic Period
Money Market	3 Month Treasury returns	1955.12 – 2003.12
U.S. ITGVT	U.S. Intermediate Term Government Bonds	1955.12 – 2003.12
U.S. LTCORP	U.S. Long Term Corporate Bonds	1955.12 – 2003.12
Fixed Income	65% ITGVT + 35% LTCORP	n/a
Balanced Allocation	60% Diversified Equity + 40% Fixed Income	n/a
Diversified Large Cap U.S. Equity	S&P500 Total Return Index	1955.12 – 2003.12
Diversified International Equity	MSCI-EAFE $USD Total Return Index	1969.12 – 2003.12
Intermediate Risk Equity	U.S. Small Capitalization Index	1955.12 – 2003.12
Aggressive Equity	Emerging Markets, NASDAQ, Hang Seng	1984.12 – 2003.12

Model Descriptions and Notes

- Pseudo-random numbers are generated using the Mersenne Twister algorithm[3]. Variance reduction techniques were not used.

- All models use a one-month interval as the time increment (i.e., 12 periods per year).

- For simplicity, the correlation matrix is constant.

- The random "shocks" that drive changes in interest rates are independent (i.e., zero correlation) of all other model factors, including equity and bond returns.

- Additional scenarios may be created by blending the accumulation factors for the market proxies.

- The Treasury yields can be used to project credited rates on fixed accounts and annuity purchase rates under GMIB options. The Treasury yields are also suitable for testing C-3 Phase I RBC (interest rate risk) on the assets and liabilities underlying the fixed account. See the section entitled "U.S. Treasury Yields" for further information.

- Bond index returns (Money Market, U.S. ITGVT, and U.S. LTCORP) are modeled as a function of interest rates. See later under "Bond Index Returns".

- Equity returns are generated from a monthly stochastic log volatility model. Additional details are given in the section "Equity Market Returns". The S&P500 TR scenarios (Diversified Equity) satisfy the calibration criteria within sampling error.

- Parameters for "Diversified International Equity" are estimated from data denominated in $US currency.

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[3] The Mersenne Twister is a well documented and robust algorithm with extremely high periodicity.

- The hypothetical "Aggressive/Exotic Equity" index was constructed by blending returns for the NASDAQ, Emerging Markets and Hang Seng indices (weights are approximately 12.5%, 25% and 62.5% respectively). This proxy has an unconditional annualized volatility of approximately 25%. This market mix is not meant to suggest a representative asset profile for this class, but used merely to build an historic index with high volatility and sufficient history.

- The estimation of the SLV model parameters for "U.S. Diversified Equity" imposed some subjective restrictions to ensure an unconditional expected total annualized return of 8.75% effective. Appendix 2 of the LCAS report documents the development of the SLV model and the associated calibration criteria.

- Parameters for the other equity markets were determined by "constrained" maximum likelihood estimation whereby the Sharpe ratio y must satisfy the following relationship:

$$y^* \le y \le 1.05 \times y^*, \quad where \ y = \frac{E[R]-r}{s}$$

$E[R]$ and s are respectively the annualized unconditional expected return (effective) and standard deviation (volatility) of market returns. For simplicity, we assume that the risk-free rate $r = 5.25\%$ (annual effective) for all markets, roughly equal to the average 3-month U.S. Treasury yield over the past 50 years. $y^* = 0.232$ is the Sharpe ratio for the S&P500 Total Return index (1955.12 – 2003.12).

U.S. Treasury Yields

The U.S. Treasury yields are generated using the "C-3 Phase I" interest rate model designed by the American Academy of Actuaries' C-3 Subgroup of the Life Risk Based Capital Task Force. The model simulates Treasury bond yields according to a stochastic variance process with mean reversion under the real-world[4] probability measure. For full details, please see "*Appendix III – Technical Aspects of the Scenario Generator and the Scenario Selection Process*" in the October 1999 report to the NAIC Life RBC Working Group.

The starting yield curve[5] (average of rates for October 2005) for the model is given in Table 3. All rates are expressed as semi-annual bond equivalent yields. Interest rate movements are uncorrelated[6] with other model factors. Before using the integrated set of prepackaged scenarios, the actuary is usually prudent to confirm that an assumption of independence is reasonable and does not lead to a material understatement of the risk exposure.

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[4] The interest rate model is designed for cash flow projections only. It is not arbitrage-free and could give inappropriate values if used to price options and other derivatives as part of an asset/liability management strategy.

[5] These values are taken from the database maintained by the U.S. Federal Reserve Bank. See http://www.federalreserve.gov/releases/h15/data.htm The 30-year maturity was estimated from historic yield curve relationships. It is not used in the modeling. The 30-year maturity could also be estimated using the "extrapolation factor" provided by the U.S. Treasury Department. See www.treas.gov/offices/domestic-finance/debt-management/interest-rate/ltcompositeindex.html. Under this approach, the 30-year maturity would be 4.64%

[6] The historic data tend to suggest a weak negative correlation between interest rate movements and equity returns.

Table 3: Starting U.S. Treasury Yield Curve – October 2005 Semi-Annual Bond Equivalent Yields

Maturity (in years)	0.25	0.5	1	2	3	5	7	10	20	30
S/A Yield	3.79%	4.13%	4.18%	4.27%	4.29%	4.33%	4.38%	4.46%	4.74%	4.85%

Bond Index Returns

The nominal monthly returns on money market and fixed income (e.g., bond) investments are generated according to:

$$r_t = b_0 \times \left(i_{t-1}^m + k\right) - b_1 \times \left(i_t^m - i_{t-1}^m\right) + s \cdot \sqrt{i_{t-1}^m} \cdot Z_t$$

where Z_t is a standard normal variate and i_t^m is the m-year Treasury yield in period t. The money market process is a function of the 3-month yield, while the ITGVT and LTCORP bond index returns are respectively modeled from the 7-year and 10-year maturities. The model parameters are provided in Table 4.

The Fixed Income scenarios (FIXED.csv) assume a constant asset mix of 65% ITGVT and 35% LTCORP. U.S. ITGVT and U.S. LTCORP are published indices (not managed funds) which respectively represent diversified portfolios of intermediate-term government bonds and long-term investment grade corporate bonds.

Table 4: Model Parameters for Bond Index Returns

	m	b_0	k	b_1	s
Money Market	0.25	0.083333	–0.00445	–0.07148	0.00370
U.S. ITGVT	7	0.083333	–0.00153	3.65043	0.05239
U.S. LTCORP	10	0.083333	0.00704	5.81293	0.08282

Although this is an empirical model, it has a plausible (and intuitive) interpretation and fits the observed data extremely well. Consistent with our expectations, the monthly return is composed of three elements:

- An income component, equal to one-twelfth of the reference yield plus a "credit/liquidity spread".
- A price movement term, equal to the duration of the index multiplied by the net increase in interest rates.
- A random shock, which reflects the relative level of interest rates and other extraneous factors.

Re-generating Interest Rate Scenarios and Bond Index Returns

As documented above, the projection of interest rates – and consequently, bond index returns – depend on the starting yield curve. As such, the company may wish to re-generate these scenarios based on the prevailing term structure (i.e., the U.S. Treasury curve at or near the reporting date, rather than October 2005). To that end, we have provided a more flexible version of the "C-3 Phase I RBC" interest rate generator in the workbook "*RbcC3Scn 2005-10-30.xls*" (see above under "U.S. Treasury Yields" for reference material). The model and parameters are identical to the original (October 1999) C-3 Phase I RBC interest rate generator, but the following points should be kept in mind:

- Unlike the original workbook, this version allows the user to send the generated scenarios directly to *.csv* text files (comma-separated values). The option to write to the open workbook is also available. If "Excel" is selected as the *Output Target* in the *Model Settings* (rather than "Comma Delimited File"), the maximum number of scenarios is approximately [$65000 \div (5 + 30 \times F)$], where F is the model frequency (e.g. 12 for monthly). An error will occur if the user selects a larger number of scenarios.

- The initial term structure is specified as the nominal semi-annual yields on U.S. Treasury bonds. All ten maturities (3m, 6m, 1, 2, 3, 5, 7, 10, 20 and 30 year) must be supplied. If the 30-year yield is unavailable or not needed, its starting value can be set equal to the 20-year rate.

- Bond index returns can be simultaneously generated based on the interest rate scenarios by setting the field *Generate Bond Indices?* to "TRUE". In this case, the user must also specify the folder (path) and filenames (*.csv* text files) for the random normal samples supplied with the pre-packaged scenarios (see Table 1B). The model and parameters are given in the section "Bond Index Returns" (see above). If "FALSE" is selected, it is the actuary's responsibility to ensure that returns on fixed income investments are consistent with the interest rate scenarios.

- Equity Market Returns

The equity return scenarios are generated from a monthly SLV model wherein the natural logarithm of the annualized volatility follows a strong mean-reverting stochastic process and the annualized drift[7] is a deterministic quadratic function of volatility. This model is not prescribed or 'preferred' above others, but was chosen because it captures many of the dynamics observed in the equity market data, including:

- Negative skewness and positive kurtosis ("fat tails") over short holding periods;
- Time-varying volatility and volatility clustering; and
- Increased volatility in bear markets.

The monthly SLV model is governed by the equations shown in Table 5. The model parameters (including descriptions) for each equity market are given in Table 6.

Table 5: SLV Model Processes

$$\tilde{v}(t) = Min\left[v^+, (1-\boldsymbol{f}) \times v(t-1) + \boldsymbol{f} \times \ln(\boldsymbol{t})\right] + \boldsymbol{s}_v \times {}_vZ_t$$

$$v(t) = Max\left\{v^-, Min\left[v^*, \tilde{v}(t)\right]\right\}$$

$$\boldsymbol{m}(t) = A + B \times \boldsymbol{s}(t) + C \times \boldsymbol{s}^2(t)$$

$$\ln\left[\frac{S(t)}{S(t-1)}\right] = \frac{\boldsymbol{m}(t)}{12} + \frac{\boldsymbol{s}(t)}{\sqrt{12}} \times {}_sZ_t$$

$S(t) = $ stock index level at time t

$v(t) = $ natural logarithm of annualized volatility in month t

$\boldsymbol{s}(t) = $ annualized volatility of stock return process in month $t = \exp\left[v(t)\right]$

$\boldsymbol{m}(t) = $ mean annualized log return ("drift") in month t

$v^- = $ lower bound for log volatility $= \ln\boldsymbol{s}^-$

$v^+ = $ upper bound for log volatility (before random component) $= \ln\boldsymbol{s}^+$

$v^* = $ absolute upper bound for log volatility $= \ln\boldsymbol{s}^*$

${}_vZ_t$ and ${}_sZ_t$ are correlated standard normal random samples, where ${}_vZ_t$ is the random "shock" to the log volatility process and ${}_sZ_t$ is the random component for the stock index return. As noted previously, the drift term $\boldsymbol{m}(t)$ is a *deterministic* quadratic function of $\boldsymbol{s}(t)$. In Table 6, $v^- = \ln\boldsymbol{s}^-$, $v^+ = \ln\boldsymbol{s}^+$ and $v^* = \ln\boldsymbol{s}^*$. From the foregoing, it should be clear that given $\boldsymbol{s}(t)$, the log (i.e., continuous) returns in any month are normally distributed with mean $\dfrac{\boldsymbol{m}(t)}{12}$ and standard deviation $\dfrac{\boldsymbol{s}(t)}{\sqrt{12}}$.

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[7] In this document, the term "drift" refers to the expected instantaneous return.

Table 6: Stochastic Log Volatility Model Parameters (Monthly) – Equity Markets

	Description	U.S.	INTL	SMALL	AGGR
t	Long-run target volatility (annualized)	0.12515	0.14506	0.16341	0.20201
f	Strength of mean reversion	0.35229	0.41676	0.3632	0.35277
s_v	Standard deviation of the log volatility process (monthly)	0.32645	0.32634	0.35789	0.34302
r	Correlation co-efficient between $_vZ_t$, $_sZ_t$	−0.2488	−0.1572	−0.2756	−0.2843
A	Drift of stock return process as $s(t) \to 0$ (i.e., intercept)	0.055	0.055	0.055	0.055
B	Co-efficient of quadratic function for $m(t)$	0.56	0.466	0.67	0.715
C	Co-efficient of quadratic function for $m(t)$	−0.9	−0.9	−0.95	−1
$s(0)$	Starting volatility (annualized)	0.1476	0.1688	0.2049	0.2496
s^-	Minimum volatility (annualized)	0.0305	0.0354	0.0403	0.0492
s^+	Maximum volatility (annualized), before random component	0.3	0.3	0.4	0.55
s^*	Maximum volatility (annualized), after random component	0.7988	0.4519	0.9463	1.1387

Table 7 provides some statistics for the simulated ("model") annualized volatilities compared to the historic data. For each market, an historic "volatility series" was constructed by calculating the 6-month rolling standard deviations of the monthly log returns. Table 8 shows statistics for the monthly log returns[8].

Table 7: Statistics for Annualized Volatility

	U.S. Equity Diversified		International Diversified Equity		Intermediate Risk Equity		Aggressive / Exotic Equity	
	History	Model	History	Model	History	Model	History	Model
Minimum	0.014	0.031	0.042	0.035	0.041	0.040	0.071	0.049
10TH Percentile	0.068	0.072	0.080	0.087	0.088	0.090	0.113	0.114
25TH Percentile	0.093	0.094	0.103	0.111	0.117	0.120	0.136	0.150
Median	0.122	0.125	0.146	0.145	0.162	0.164	0.177	0.202
75TH Percentile	0.164	0.167	0.194	0.190	0.213	0.224	0.280	0.274
90TH Percentile	0.200	0.217	0.237	0.243	0.292	0.296	0.360	0.360
Maximum	0.408	0.799	0.398	0.452	0.584	0.946	0.748	1.139
Average	0.133	0.137	0.153	0.157	0.181	0.182	0.221	0.224
Standard Deviation	0.062	0.062	0.064	0.065	0.094	0.089	0.125	0.106
Skewness	1.436	1.426	0.694	1.175	1.672	1.544	1.999	1.505
Kurtosis	3.810	3.756	0.496	1.980	3.902	4.242	5.161	4.090

Table 8: Statistics for Monthly Log Returns

	U.S. Equity Diversified		International Diversified Equity		Intermediate Risk Equity		Aggressive / Exotic Equity	
	History	Model	History	Model	History	Model	History	Model
0.1TH Percentile	–0.2423	–0.2199	–0.1553	–0.2268	–0.3452	–0.3219	–0.4541	–0.3944
10TH Percentile	–0.0451	–0.0447	–0.0519	–0.0518	–0.0577	–0.0612	–0.0662	–0.0769
25TH Percentile	–0.0155	–0.0156	–0.0176	–0.0197	–0.0212	–0.0211	–0.0190	–0.0275
Median	0.0110	0.0086	0.0103	0.0081	0.0148	0.0105	0.0164	0.0119
75TH Percentile	0.0374	0.0309	0.0385	0.0345	0.0452	0.0391	0.0575	0.0473
90TH Percentile	0.0549	0.0540	0.0637	0.0619	0.0783	0.0694	0.0899	0.0842
99.9TH Percentile	0.1533	0.1691	0.1644	0.1953	0.2443	0.2258	0.2112	0.2707
Average	0.0084	0.0060	0.0086	0.0062	0.0112	0.0063	0.0120	0.0065
Standard Deviation	0.0426	0.0436	0.0487	0.0492	0.0598	0.0590	0.0721	0.0724
Skewness	–0.59	–0.67	–0.33	–0.40	–0.72	–0.89	–1.47	–0.91
Kurtosis	2.42	4.02	0.78	2.69	4.16	5.33	7.84	5.20

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[8] In Table 8, the 0.1% and 99.9% values for history are respectively the minimum and maximum monthly returns over the observation period

Correlations

The correlation matrix governing the stochastic processes is given in Table 9. The matrix is decomposed using the standard technique of Cholesky[9] factorization in order to generate correlated normal samples.

Table 9: Correlation Matrix for Integrated Scenario Model

	US LogVol	US LogRet	INTL LogVol	INTL LogRet	SMALL LogVol	SMALL LogRet	AGGR LogVol	AGGR LogRet	MONEY Return	ITGVT Return	LTCORP Return
US LogVol	1	−0.249	0.318	−0.082	0.625	−0.169	0.309	−0.183	0.023	0.075	0.080
US LogRet	−0.249	1	−0.046	0.630	−0.123	0.829	−0.136	0.665	−0.120	0.192	0.393
INTL LogVol	0.318	−0.046	1	−0.157	0.259	−0.050	0.236	−0.074	−0.066	0.034	0.044
INTL LogRet	−0.082	0.630	−0.157	1	−0.063	0.515	−0.098	0.558	−0.105	0.130	0.234
SMALL LogVol	0.625	−0.123	0.259	−0.063	1	−0.276	0.377	−0.180	0.034	0.028	0.054
SMALL LogRet	−0.169	0.829	−0.050	0.515	−0.276	1	−0.142	0.649	−0.106	0.067	0.267
AGGR LogVol	0.309	−0.136	0.236	−0.098	0.377	−0.142	1	−0.284	0.026	0.006	0.045
AGGR LogRet	−0.183	0.665	−0.074	0.558	−0.180	0.649	−0.284	1	0.034	−0.091	−0.002
MONEY Return	0.023	−0.120	−0.066	−0.105	0.034	−0.106	0.026	0.034	1	0.047	−0.028
ITGVT Return	0.075	0.192	0.034	0.130	0.028	0.067	0.006	−0.091	0.047	1	0.697
LTCORP Return	0.080	0.393	0.044	0.234	0.054	0.267	0.045	−0.002	- 0.028	0.697	1

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[9] The Cholesky decomposition method works when the matrix is positive semi-definite (i.e., has non-negative eigenvalues). The correlation matrix in Table 9 satisfies this property.

Tables 10 and 11 respectively show the historic ("observed") and sample (from the 10,000 prepackaged scenarios) correlations for the monthly log returns.

Table 10: Historic Correlations for Monthly Log Returns

	S&P500	MSCI-EAFE $USD	U.S. SmallCap	Aggressive Equity	Money Market	U.S. ITGVT	U.S. LTCORP
S&P500	1						
MSCI-EAFE $USD	0.560	1					
U.S. SmallCap	0.759	0.447	1				
Aggressive Equity	0.595	0.488	0.579	1			
Money Market	−0.046	−0.059	−0.053	0.002	1		
U.S. ITGVT	0.137	0.091	0.042	−0.064	0.113	1	
U.S. LTCORP	0.280	0.171	0.184	−0.005	0.026	0.822	1

Table 11: Model Correlations (10,000 Scenarios) for Monthly Log Returns

	U.S.	INTL	SMALL	AGGR	MONEY	ITGVT	LTCORP
U.S.	1						
INTL	0.558	1					
SMALL	0.762	0.445	1				
AGGR	0.577	0.481	0.565	1			
MONEY	−0.037	−0.032	−0.032	0.010	1		
ITGVT	0.145	0.100	0.049	−0.067	0.080	1	
LTCORP	0.304	0.185	0.202	−0.002	0.011	0.774	1

Scenario Statistics – Accumulation Factors

Tables 12 through 15 provide sample statistics for the accumulation factors (gross wealth ratios) over various holding periods. For reference, the calibration points[10] (applicable to U.S. Diversified Equity) are also shown. Within sampling error, the statistics for the U.S. Equity scenarios satisfy these criteria. The statistics are calculated across all 10,000 scenarios over the given horizon, starting five (5) years into the projection.

Table 12: Statistics for 1-Year Accumulation Factors

	Calibration Point	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
0.5%		1.000	0.914	0.836	0.666	0.644	0.553	0.449
1.0%		1.001	0.926	0.864	0.711	0.696	0.619	0.513
2.5%	**0.78**	1.003	0.945	0.897	0.779	0.755	0.697	0.611
5.0%	**0.84**	1.007	0.962	0.923	0.832	0.807	0.758	0.683
10.0%	**0.90**	1.013	0.980	0.950	0.888	0.866	0.835	0.782
50.0%		1.040	1.045	1.053	1.085	1.081	1.093	1.097
90.0%	**1.28**	1.072	1.121	1.167	1.285	1.328	1.371	1.447
95.0%	**1.35**	1.082	1.147	1.206	1.353	1.410	1.466	1.569
97.5%	**1.42**	1.091	1.171	1.243	1.407	1.490	1.562	1.697
99.0%		1.104	1.202	1.288	1.494	1.593	1.679	1.878
99.5%		1.113	1.222	1.312	1.560	1.670	1.758	2.008
Mean		1.042	1.048	1.057	1.086	1.092	1.100	1.110
Stdev		0.023	0.057	0.088	0.160	0.186	0.216	0.274

Table 13: Statistics for 5-Year Accumulation Factors

	Calibration Point	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
0.5%		1.015	0.943	0.824	0.560	0.527	0.395	0.281
1.0%		1.022	0.970	0.864	0.628	0.579	0.473	0.334
2.5%	**0.72**	1.040	1.009	0.925	0.724	0.670	0.577	0.446
5.0%	**0.81**	1.060	1.044	0.983	0.820	0.763	0.677	0.553
10.0%	**0.94**	1.089	1.085	1.046	0.933	0.879	0.816	0.703
50.0%		1.224	1.261	1.306	1.455	1.458	1.479	1.498
90.0%	**2.17**	1.412	1.511	1.667	2.200	2.379	2.583	2.964
95.0%	**2.45**	1.481	1.610	1.808	2.467	2.726	3.021	3.600
97.5%	**2.72**	1.550	1.706	1.957	2.707	3.035	3.451	4.193
99.0%		1.643	1.832	2.121	3.109	3.476	3.988	5.024
99.5%		1.727	1.920	2.285	3.385	3.818	4.485	5.695
Mean		1.242	1.285	1.339	1.523	1.564	1.622	1.708
Stdev		0.132	0.179	0.261	0.516	0.617	0.750	0.990

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[10] The calibration points are taken from Table 1 in Appendix 2 of the March 2005 LCAS Report.

Table 14: Statistics for 10-Year Accumulation Factors

	Calibration Point	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
0.5%		1.067	1.032	0.923	0.565	0.496	0.332	0.231
1.0%		1.090	1.081	0.975	0.650	0.563	0.409	0.299
2.5%	**0.79**	1.125	1.145	1.069	0.797	0.702	0.557	0.413
5.0%	**0.94**	1.171	1.205	1.154	0.935	0.853	0.703	0.552
10.0%	**1.16**	1.231	1.275	1.255	1.109	1.038	0.918	0.763
50.0%		1.515	1.623	1.741	2.077	2.118	2.187	2.226
90.0%	**3.63**	2.009	2.217	2.559	3.780	4.178	4.890	5.927
95.0%	**4.36**	2.226	2.484	2.898	4.470	5.103	6.077	7.983
97.5%	**5.12**	2.439	2.742	3.284	5.168	5.947	7.284	9.828
99.0%		2.766	3.152	3.828	6.226	7.323	8.978	12.873
99.5%		3.021	3.374	4.224	6.879	8.333	10.427	16.278
Mean		1.583	1.704	1.849	2.314	2.437	2.620	2.960
Stdev		0.344	0.419	0.580	1.153	1.412	1.787	2.616

Table 15: Statistics for 20-Year Accumulation Factors

	Calibration Point	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
0.5%		1.266	1.309	1.286	0.721	0.619	0.374	0.216
1.0%		1.313	1.399	1.378	0.841	0.738	0.493	0.291
2.5%		1.402	1.545	1.549	1.115	0.966	0.692	0.472
5.0%	**1.51**	1.501	1.674	1.740	1.399	1.234	0.968	0.701
10.0%	**2.10**	1.645	1.857	1.960	1.787	1.637	1.387	1.069
50.0%		2.389	2.822	3.270	4.278	4.463	4.648	4.777
90.0%	**9.02**	4.109	5.040	6.134	10.119	11.450	14.408	19.852
95.0%	**11.70**	4.883	6.152	7.635	13.015	15.413	19.961	29.071
97.5%		5.854	7.297	9.385	15.943	19.895	26.019	41.504
99.0%		7.481	9.193	11.921	19.819	26.095	35.399	60.593
99.5%		8.850	11.144	14.336	23.325	30.479	45.713	77.272
Mean		2.720	3.242	3.808	5.336	5.894	6.797	8.749
Stdev		1.367	1.736	2.275	3.968	5.235	7.216	13.259

Scenario Statistics – Returns

Tables 16 through 18 show statistics for the gross returns on each asset class over various holding periods (i.e., years 1 – 10, 11 – 20, etc.). As expected, the returns (and volatilities) on the fixed income funds rise over time, consistent with the parameterization of the interest rate model. The returns and volatilities on the equity funds are relatively constant (within sampling error) since the starting volatility (at "time zero" – see Table 6) for each market is set roughly equal to the long-term average.

Table 16: Average Annualized Holding Period Return (Annual Effective)

	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
1 – 10	4.22%	4.63%	5.32%	8.79%	9.35%	10.17%	11.45%
11 – 20	5.04%	6.06%	6.98%	8.76%	9.26%	10.09%	11.45%
21 – 30	5.41%	6.65%	7.65%	8.71%	9.24%	10.04%	11.40%
30 Years	5.10%	5.91%	6.67%	8.76%	9.29%	10.14%	11.40%

Table 17: Median Annualized Holding Period Return (Annual Effective)

	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
1 – 10	3.99%	4.43%	5.03%	7.65%	7.81%	8.16%	8.30%
11 – 20	4.44%	5.29%	6.10%	7.69%	7.80%	8.07%	8.44%
21 – 30	4.73%	5.68%	6.47%	7.62%	7.78%	8.05%	8.45%
30 Years	4.42%	5.23%	5.96%	7.54%	7.74%	7.92%	8.18%

Table 18: Annualized Volatility

	MONEY	ITGVT	LTCORP	U.S.	INTL	SMALL	AGGR
1 – 10	0.68%	4.76%	7.47%	15.15%	17.06%	20.48%	25.18%
11 – 20	0.85%	5.24%	8.17%	15.11%	17.04%	20.44%	25.10%
21 – 30	0.90%	5.43%	8.46%	15.11%	17.06%	20.48%	25.08%
30 Years	0.83%	5.15%	8.04%	15.10%	17.03%	20.44%	25.08%

Pre-packaged Scenario Files

The pre-packaged scenarios are available for download from the American Academy of Actuaries' website (http://www.actuary.org/life/phase2.asp). The files are provided in comma-separated value (*.csv*) text format (i.e., each scenario is terminated with a new line and line feed character).

It is important to note that the scenarios have been constructed so that the Kth scenario (path) for each asset class *must* be used together and considered one "future investment return environment." It is inappropriate to misalign the ordering of scenarios (e.g., scenario J for "Diversified U.S. Equity" cannot be combined with scenario K for "Diversified International Equity", $J \neq K$).

The scenario files provide monthly values for 30 years. The first column applies at "time zero". For interest rates, the first column in the scenario matrix is the yield at the start of the test. For all other files, the first column is composed entirely of ones (certain software programs may require this column to be removed before use). Hence, the size of the 'scenario matrix' in each file is $10,000 \times (1 + 12 \times 30) = 10,000 \times 361$.

The U.S. Treasury yields are expressed as nominal semi-annual bond equivalent yields in decimal format. All other returns are expressed as periodic (not cumulative) market accumulation factors (i.e., monthly "gross wealth ratios"). Interest rates are assumed to change at the start of each month, hence the value in column T applies for month T−1. The market accumulation factor in column T represents the growth in month T−1.

Durations 0 through 9 (inclusive) for the first ten (10) scenarios in the Diversified U.S. Equity file are shown below in Table 19. Table 20 gives the first ten scenarios for the 10-year U.S. Treasury yields. Using scenario 6 as an example, the yield in month 3 is 4.8507% and the monthly U.S. equity growth factor is 1.006528.

Table 19: First 10 Scenarios for Diversified U.S. Equity (US.csv) – Monthly Accumulation Factors

		0	1	2	3	4	5	6	7	8	9
1		1	1.092469	1.083986	1.006021	1.015863	1.020886	1.017517	1.033370	1.065896	1.014492
2		1	0.919721	0.974064	1.004553	1.010235	1.013128	0.996171	1.008690	0.979059	1.001054
3		1	1.017425	1.019115	1.026518	0.975530	1.006764	1.020923	0.956538	0.963677	0.985936
4		1	1.050778	1.009128	1.083217	0.956835	0.968311	1.042744	0.983705	1.045328	1.049569
5		1	0.980780	0.994217	1.019358	0.991551	1.120387	0.877891	1.049358	0.950271	1.005277
6		1	0.971779	0.978151	1.006528	1.005762	1.060561	1.058023	1.014724	0.972309	0.993515
7		1	0.925176	1.009579	1.062304	1.019200	1.050946	1.045153	1.015894	1.001342	1.000698
8		1	1.050359	0.995239	0.990647	1.013092	1.000848	1.004970	0.974614	1.024655	0.991690
9		1	1.035438	1.010034	1.009361	1.013842	1.022945	1.015931	1.020676	0.920945	1.063518
10		1	1.018519	1.162125	1.030109	1.052653	1.070173	1.025596	1.010738	0.958566	0.978353

Table 20: First 10 Scenarios for 10-year U.S. Treasury yield (UST_10y.csv) – Nominal S/A BEY

		0	1	2	3	4	5	6	7	8	9
1		0.0446	0.046138	0.046979	0.047191	0.047966	0.049927	0.048230	0.048098	0.051619	0.052981
2		0.0446	0.046985	0.047156	0.048724	0.050417	0.052102	0.052082	0.050493	0.049428	0.049755
3		0.0446	0.045632	0.044629	0.046042	0.046409	0.045327	0.043633	0.041716	0.040803	0.042023
4		0.0446	0.046484	0.045296	0.045724	0.045813	0.045002	0.045640	0.047096	0.047263	0.045526
5		0.0446	0.047029	0.046476	0.045087	0.042563	0.038429	0.036319	0.037115	0.037947	0.038242
6		0.0446	0.046450	0.047356	0.048507	0.048445	0.046922	0.045863	0.047233	0.046214	0.044602
7		0.0446	0.047670	0.048973	0.049932	0.050239	0.047696	0.046144	0.047872	0.049243	0.049774
8		0.0446	0.046002	0.044207	0.043409	0.041838	0.040434	0.038774	0.038911	0.042185	0.043620
9		0.0446	0.045730	0.046652	0.048591	0.049629	0.052189	0.052902	0.054083	0.057010	0.058027
10		0.0446	0.045689	0.044358	0.046050	0.046862	0.046160	0.045724	0.045432	0.046704	0.047710

Scenario Conversion

It will be the company's responsibility to translate the scenarios into the format required by its projection software. This could require a change to the periodicity (i.e., timestep) and/or type of return. This conversion will normally be straightforward, but is clearly a critical step in the scenario testing. The actuary will usually conduct or oversee the successful conversion of the scenarios. For conversion purposes, the company should assume geometric averaging to transform the interest rates.

Tables 21 and 22 show the conversion to a quarterly timestep for a variety of return types for scenario 6.

Table 21: Conversion of Diversified U.S. Equity Accumulation Factors to Quarterly Periodicity

Month	Accumulation Factor	Quarter	Accumulation Factor (1)	Log Return (2)	Nominal Return (3)
1	0.971779				
2	0.978151	1	0.956752	−0.044211	−0.043248
3	1.006528				
4	1.005762				
5	1.060561	2	1.128563	0.120946	0.128563
6	1.058023				

(1) Product of the monthly accumulation factors.
(2) Natural logarithm of the quarterly accumulation factor = sum of monthly log returns.
(3) Quarterly accumulation factor − 1.

Table 22: Conversion of 10-year U.S. Treasury Yields to Quarterly Periodicity

Month	Semi-Annual Bond Yield	Quarter	Semi-Annual Bond Yield (4)	Effective Rate (5)	Continuous Return (6)
1	0.046450				
2	0.047356	1	0.0474375	0.0480001	0.0468837
3	0.048507				
4	0.048445				
5	0.046922	2	0.0470764	0.0476304	0.0465309
6	0.045863				

(4) For the kth quarter, $i_k^* = 2 \times \left[\left(\prod_{t=1}^{3} \left(1 + \frac{i_{3(k-1)+t}}{2} \right) \right)^{\frac{1}{3}} - 1 \right]$.

(5) $i_k' = \left(1 + \frac{i_k^*}{2} \right)^2 - 1$.

(6) Natural logarithm of $(1 + i_k')$.

Representative Scenarios

Recognizing that a company may wish to run fewer than 10,000 scenarios, a utility in Microsoft® Excel 2002 has been created which selects a representative subset of size *N* (user-specified). The workbook "*Scenario Picking Tool (AAA LCAS C3 Phase II RBC) v7 Locked.xls*" is available for download from the AAA website.

A significance measure *S* (defined below) has been calculated for each scenario. The selection process ranks (orders) the significance values and stratifies the sample, picking the mid-point of each stratum as the representative. The selection process has been designed so that the representative scenarios are equally likely.

The significance *S* of an investment return path is defined as:

$$S = \sqrt{\sum_{t=1}^{H}\left(\prod_{k=1}^{t}\frac{1}{AF_k}\right)^2}$$

where time is measured in months and AF_k is the accumulation factor in month k. We have selected a horizon $H = 180$ months (i.e., 15 years) in the calculations.

The workbook is fully documented and easy to use. Only a handful of fields (cells) are accessible; the rest of the workbook is locked for protection. Input fields are identified by light yellow or ivory background shading and blue font for text. The actuary simply selects an asset class for the scenario selection using the drop-down list and inputs the desired number of scenarios (minimum 200). The selected asset class should closely approximate the investment option (fund category) to which the company is most exposed. For most companies, this would be the "Diversified U.S. Equity" class.

The representative scenarios are identified by scenario number. The significance measures are also provided. This output is located in the cells with light green shading and may be copied and pasted into other applications.

Sampling Error

Statistical measures estimated by simulation are always subject to sampling error. For tail measures (e.g., quantiles, CTE, etc.) the mis-estimation due to sampling error can be significant, especially when there are fewer than 1000 scenarios. As such, the selection method to obtain representative scenarios should be used with caution. As a general rule, the company should attempt to process as many scenarios as possible. If fewer than 1000 scenarios are used, it is strongly advised that the company investigate the potential sampling error and adjust the results according to prior sensitivity testing. Such analysis could include testing the variability in results for a small, suitably chosen "representative portfolio."[11]

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[11] Suppose the company wishes to use *N* scenarios ($N < 10{,}000$). It could adjust the total portfolio CTE(*X*) based on the difference in CTE(*X*) results for a representative portfolio run over (1) all 10,000 pre-packaged scenarios and (2) the selected subset of *N* paths.