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Dear Mark and Hans:

The topic of discounting plays a key role in several conceptual accounting projects that are currently underway. The concept of "fair value" is also prominent in these discussions. A wide variety of methods have been, and continue to be, developed for discounting in the context of fair value measurements. Such methods combine both discounting and provision for the market price of risk in various ways. Concern has arisen among many actuaries, however, that when certain methods are specifically enumerated in accounting standards, it could imply a prohibition on the use of alternative methods that are consistent with stated valuation objectives, unless it is clearly stated that alternative methods are allowed.

As accounting standards evolve, the need to allow the use of alternative methods that are consistent with a set of stated principles could be overlooked. The Financial Reporting Committee of the American Academy of Actuaries[1] ("Academy") wishes to emphasize the importance of allowing flexibility in methodology for discounting and fair value measurement, subject to stated principles and valuation objectives. With that in mind, we have developed a White Paper, "Notes on the Use of Discount Rates in Accounting Present Value Estimates," which is attached for your information.

Current accounting standards provide some comfort with respect to the use of alternative methods. For example, Statement of Financial Accounting Concepts No. 7 ("CON 7") says, in paragraph 57:

> "In recent years, financial institutions and others have developed and implemented a variety of pricing tools designed to estimate the fair value of assets and liabilities. It is not possible here to describe all of the many (often proprietary) pricing models currently in use. However, those tools often build on concepts similar to those outlined in this Statement as well as other developments in modern finance, including option pricing and similar models. For example, the well-known Black-Scholes option pricing model uses the elements of a fair value measurement described in paragraph 23 as appropriate in estimating the fair value of an option. To the extent that a pricing model includes each of the elements of fair value, its use is consistent with this statement."

[1] The American Academy of Actuaries ("Academy") is a 16,000-member professional association whose mission is to serve the public on behalf of the U.S. actuarial profession. The Academy assists public policymakers on all levels by providing leadership, objective expertise, and actuarial advice on risk and financial security issues. The Academy also sets qualification, practice, and professionalism standards for actuaries in the United States

Similarly, Statement of Financial Accounting Standards No. 157 ("FAS 157") includes the following statement in Appendix B:

> "This appendix neither prescribes the use of one specific present value technique nor limits the use of present value techniques to measure fair value to the techniques discussed herein."

Nearly identical wording appears in the current IASB Exposure Draft on Fair Value Measurement ("IASB ED 2009/5") in the first paragraph of Appendix C.

Our White Paper describes a number of valuation methods commonly used today that are consistent with the principles stated in CON 7, FAS 157, and IASB ED 2009/5. Several of the methods discussed in the White Paper are not explicitly mentioned in those documents. These include: certain methods for valuation of insurance liabilities with non-guaranteed investment elements; stochastic methods that use adjusted probabilities (rather than adjusted cash flows or adjusted discount rates); and stochastic methods that use discount rates that vary by scenario.

Note that we do not advocate adding descriptions of these methods to future accounting standards. Instead, we hope that this sampling of currently applied methodologies will reinforce the need for future standards to continue the practice of explicitly stating that alternate approaches are allowed.

If we can be of further assistance, please contact the Academy's Senior Risk Management and Financial Reporting Policy Analyst, Tina Getachew, at getachew@actuary.org or +1 202.223.8196.

Sincerely yours,

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Vice-Chair, Financial Reporting Committee
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Cc: Sam Gutterman (Chair, Insurance Accounting Committee, International Actuarial Association)

AMERICAN ACADEMY *of* ACTUARIES

Discussion on the use of Discount Rates in Accounting Present Value Estimates

American Academy of Actuaries[1]
Financial Reporting Committee

September 2009

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The members of the Discounting Subgroup that are responsible for this white paper are as follows:

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Introduction and Purpose

At the time of writing (mid-2009), several conceptual accounting projects were underway that involve discussion of discount rates and present value estimates, including but not limited to the IASB/FASB joint project on Revenue Recognition and the IASB/FASB joint project on insurance contracts. When discount rates[1] and present value estimates have been discussed in these wider projects, the discussion has tended to be limited in scope. There is concern, however, that such limited discussions may result in accounting standards that limit the scope or breadth of techniques that are allowed when making present value estimates for accounting purposes.

In this paper, we use the term "present value *estimate*" rather than the accounting term *measurement* to make a useful distinction. Present value *estimates* involve unknown (and in many cases unknowable) future outcomes; the estimating process thus aims to determine a single value (*i.e.*, the *present value*) as representative of a range or distribution of potential future outcomes. By contrast, the term *measurement* is often used in the context of known or knowable fixed quantities. In the context of insurance, for example, the total of benefits actually paid is a measurement; the present value of benefit obligations is an estimate. Estimates are therefore needed for accounting measurement of items whose value is uncertain.

Our focus in this document is on present value estimates that are intended to reflect market conditions on the valuation date[2]. There are several terms for such estimates, including fair value, fulfillment value, current value, and so on. These estimates share the common trait that similar methods of discounting can be used for any one of them, and it is those methods of discounting that we wish to discuss.[3]

FASB Statement of Financial Accounting Concepts No. 7 ("CON7") allows a broad range of valuation techniques when it says, in paragraph 57:

> "In recent years, financial institutions and others have developed and implemented a variety of pricing tools designed to estimate the fair value of assets and liabilities. It is not possible here to describe all of the many (often proprietary) pricing models currently in use. However, those tools often build on concepts similar to those outlined in this Statement as well as other developments in modern finance, including option pricing and similar models. For example, the well-known Black-Scholes option pricing model uses the elements of a fair value measurement described in paragraph 23 as appropriate in estimating the fair value of an option. To the extent that a pricing model includes each of the elements of fair value, its use is consistent with this statement."

The elements of fair value estimate in paragraph 23 are:

"

[1] In this white paper we use the term "discount rate" as it is used in FAS 157, somewhat interchangeably with "interest rate". Texts on the theory of the time value of money draw a technical distinction between "discount rate" and "interest rate", but for ease of understanding we ignore this technical issue.

[2] The market conditions we are concerned with in this discussion of discounting are limited to market interest rates and items related to interest rates such as liquidity premiums, credit spreads, and market volatility. The definition of accounting measures such as fair value and fulfillment value sometimes differ in whether market-based or entity-specific assumptions are to be used when projecting the cash flows to be discounted, but we are not concerned with those differences in this paper.

[3] This means that amortized-cost methodologies for accounting valuation are outside the scope of this discussion.

a. An estimate of future cash flow, or in more complex cases, series of future cash flows at different times.
b. Expectations about possible variations in the amount or timing of those cash flows.
c. The time value of money, represented by the risk-free rate of interest.
d. The price for bearing the uncertainty inherent in the asset or liability.
e. Other, sometimes unidentifiable, factors including illiquidity and market imperfections."

Exactly the same elements of fair value are documented in the May 2009 IASB Exposure Draft ("IASB ED 2009/5") on *Fair Value Measurement*, in Appendix C, paragraph 2.

CON 7 was issued in February 2000. Some more recent accounting discussions that mention present values or discounting explicitly refer to the risk-free rate mentioned in paragraph 23c above, without putting it in the context of a complete valuation technique that must embody all five elements. Some readers interpret such mention as a proposed rule forbidding the use of valuation techniques which use discount rates that reflect the combined effect of several of the five elements above, including paragraph 23d (uncertainty) and paragraph 23e (illiquidity).

In addition, some accounting discussions[4] mention the "expected cash flow approach" wherein several possible patterns of future cash flow are projected and then probability weighted to obtain "expected" future cash flows, which are then discounted to obtain the present value. Some readers interpret such mention as a proposed rule forbidding the use of path-specific discount rates that are commonly used in practice when future cash flows depend on the uncertain level of future interest rates. And, some readers lament the limited nature of discussion on how the market price of risk and uncertainty is incorporated into the "expected cash flow approach".

We are concerned that these discussions and others are potentially leading to valuation rules that may not reflect all the elements of fair value outlined above. This paper explains the basic theory behind some valuation techniques that reflect all five elements of fair value as listed in CON 7 paragraph 23. It is our hope that the Boards will reaffirm the relevance of all five elements in any valuation intended to reflect market interest rates on the valuation date, and will continue to use language like the following, which appears in both FAS 157 and IASB ED 2009/5, in the respective Appendices titled *Present Value Techniques*.

> "This appendix neither prescribes the use of one specific present value technique nor limits the use of present value techniques to measure fair value to the techniques discussed herein."

Section 1 of this white paper covers the use of discount rates other than the risk-free rate. Section 2 covers stochastic techniques, where multiple paths of future cash flows are projected and probability-weighted to obtain a single value.

All of the techniques discussed in this paper would appear to be consistent with paragraph 57 of CON 7, since they reflect the five elements listed in paragraph 23. However, many of the discounting techniques discussed here differ from those specifically mentioned in FAS 157 and IASB ED 2009/5. The purpose of this paper is to explain these methods with a focus on how they reflect "the price for bearing the uncertainty inherent in the asset or liability". The intent is

[4] E.g., paragraph 46 of CON7

to clarify any discussion concerning the use of techniques like these under a broad interpretation of CON 7 paragraph 57.

Section 1 – The discount rate and provision for risk

The time value of money is conceptually represented by the risk-free rate of interest[5]. But for accounting purposes one is often required to measure the value of an asset or liability that is not risk-free. There are many ways to adjust for risk in a present value calculation. One convenient and frequently used method is to adjust the discount rate, reflecting market-observable discount rates for cash flow streams with similar timing and risk characteristics. The very name of the term "risk-free rate" presumes that there exist other rates of interest that exist when risk is present.

This concept can easily be illustrated using a simple corporate bond as an example. Our example bond has a market value of $100.00 today, and is scheduled to mature in one year for $107.00. No coupons or other cash flows will contractually occur between today and the maturity date one year from now. Assume that the risk-free rate for a one-year term is 5%, and also that this bond bears a default risk.

Since the market value of the bond is $100.00, we don't even need to pick a discount rate or evaluate the provision for risk. The market price implies that the risk-adjusted discount rate is 7.0%. However, it is instructive to examine the components that are embodied in the difference between the risk-adjusted and the risk-free rates. Note that the market's provision for risk in the price of this bond can be dissected into two parts. First there is the market's perception of the expected, or probability-weighted cost of default. Second, there is the risk of whether a default will or will not occur, and the price the market extracts for bearing that risk. In the case of a bond, it is not usually possible to separate these two elements; all that is observable is the total market price for the risky asset.

<u>Example 1: Discount rate includes full provision for risk</u>
Discount rate = 7%.

Under this method, the discount rate provides the full provision for both aspects of the risk. We discount the contractual cash flows at the market's risk-adjusted discount rate of 7%. The present value of $107.00 at 7% interest is $100.00.

<u>Example 2: Cash flows are adjusted for expected defaults; discount rate provides only for the cost of uncertainty about default.</u>
Discount rate= 5.93%

In this example, we need to make an assumption about the expected rate of default. As noted earlier, the expected rate of default cannot be separately identified by market observations. For purposes of this example only, we assume that the expected rate of

[5] As a practical matter, identifying the risk-free rate, or many of the other conceptual quantities that will be discussed in this paper, is not always easy. The purpose here is to focus on the conceptual framework that governs relationships between quantities. Estimation of the quantities themselves (such as the risk free rate) is a separate topic and is beyond the scope of this paper.

default is 1%. Under this method the discount rate provides only for the risk of whether default will occur, but does not adjust for the probability-weighted "expected" cost of default. The cash flows are adjusted to reflect the expected defaults. To adjust the cash flows, we subtract an amount reflecting 1% probability of total default, or $1.07, so the expected cash flow before discounting is $107.00 - $1.07 = $105.93[6]. Discounting the expected cash flow at 5.93% then provides the $100.00 market value. Observe that 5.93% is greater than the risk-free rate because it includes a provision for the risk of whether default will occur (that is, the assumed market price for bearing the default risk).

Example 3: Discount rate includes no provision for risk
Discount rate = 5.00%

Under this method the discount rate is the risk-free rate and all adjustment for risk is done by adjusting the cash flows. The adjustment to cash flows must include not only the $1.07 for expected defaults, but also a market-calibrated provision for the risk of whether default will occur. The cash flows must be adjusted to be "certainty equivalents". The market-calibrated provision for risk in this case is $0.93, so the certainty equivalent cash flow is $107.00 – $1.07 – $0.93 = $105.00. Discounting at the risk-free rate of 5% then leads to the market price of $100.00.

IASB ED 2009/5 lists three methods for adjusting for risk in Appendix C, paragraph C6 (a), (b), and (c). Example 1 above is analogous to the "discount rate adjustment technique" from C6(a). Example 2 is analogous to "method 2 of the expected present value technique" from C6(c). Example 3 is analogous to "method 1 of the expected present value technique" from C6(b). These same three methods are outlined in FAS 157 Appendix B.

When the purpose for a valuation is to obtain a market-based or market-consistent value for an item that includes risk or uncertainty, it is best to use the most directly applicable and readily available information concerning market pricing for similar risks. This information can take several forms, leading to several different valuation techniques for estimating the effects of risk. In the example above, the most readily available information on market pricing is the market spread over the risk-free rate, or 2% = 7% - 5%. This spread could be used as means of estimating the effect of risk when valuing other assets that have similar risk and payment characteristics. However, there are situations where cash flow adjustments are more directly related to observable market parameters than are discount rates. A variety of methods should be allowed, as the most direct method is likely to be the most reliable, and depends on the circumstances.

Sections 1.1 to 1.3 present examples of risk adjustment methods for a variety of risks that are relevant to valuing insurance contract liabilities. It is understood that insurance contracts often involve many different risks, all of which must be reflected in the same valuation, so several adjustments often need to be combined together. Insurance contracts may also require assumptions and estimates to account for the difference between contract risks and market-observable analogues used as inputs to the valuation.

[6] Or, equivalently, take a weighed average of 107 x 99% [assumed probability of payment in full] + 0 x 1% [assumed probability of total default]. This simple example assumes no partial recovery on default.

1.1 Risks of own credit standing and liquidity: the financing rate

When a customer pays money to initiate an insurance contract with an insurer, the customer takes on two risks:

- Credit risk: The risk that the insurer will not pay contractual benefits when due.
- Liquidity risk: The loss of ready access to funds. The customer loses access to their funds between the time the contract is paid for and the time when contractual benefits are due.[7]

These two risks are also present in a financing transaction wherein a lender provides funds to a borrower. The lender takes the credit risk and also loses access to the funds except under contractual terms. There is a clear analogy here – the insurance customer is in the position of the lender and the insurer is in the position of the borrower.

As was mentioned earlier, provision for risk can be made in many forms. We now discuss how to provide for the credit and liquidity risks. We start from a valuation that does not provide for risk: the present value of certain cash flows at the risk-free rate. We note that risks to the insurer should increase the value, and risks to the customer should decrease the value. Since credit risk and liquidity risk are both risks to the customer, when taken into account they should decrease the value of the insurer's liability[8].

A decrease in present value can be obtained by increasing the discount rate with some sort of interest rate spread. Let the risk-free rate be i_{rf}; let the spread for credit standing be s_{credit}; and, let the spread for liquidity risk be $s_{liquidity}$. The adjusted discount rate is then $i_{rf} + s_{credit} + s_{liquidity}$ [9].

As was noted above, these same two risks are present in a financing transaction. Therefore, one can think of this adjusted discount rate as a financing rate. For purposes of this document, we will symbolize the financing rate as: $i_f = i_{rf} + s_{credit} + s_{liquidity}$.

1.2 Risk of the uncertain level of claims

In the valuation of insurance liabilities, one of the most common risks is that of the unknown level of actual incurred claims and claim payments. A margin for this risk should increase the value of the liability so that it is greater than the discounted present value of expected claims taken at the financing rate[10].

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[7] Even under contracts that contain a deposit element wherein the deposit is accessible, generally part of the customer's funds do enter the deposit element and are used to pay for pure insurance protection. This part of the funds is subject to liquidity risk under such contracts. In addition, even when the deposit element is accessible, there is often a surrender charge that enforces a penalty for withdrawal, thereby reducing liquidity of the funds.

[8] There is considerable debate concerning whether valuation of liabilities should reflect the credit risk of the liability holder (for example, the IASB has requested comments on exactly this issue for liability measurement). We take no position on this issue in this white paper, At the time of writing, the Academy is in the process of formulating its response to IASB Discussion Paper 2009/2, *Credit Risk in Liability Measurement.*

[9] As noted earlier, in practice it may be difficult if not impossible to separately identify s_{credit} and $s_{liquidity}$, and a single market observation of a financing rate is used as a stand-in for both.

[10] This risk margin might be included as part of the reported liability or it might be shown as a separate item in the financial statements, depending on evolving rules for financial statement presentation and disclosure.

Two of the most common methods of providing this margin are:

1) adjusting the cash flows by adding to them a "cost of capital[11]" amount that represents the market price of risk; and
2) adjusting the discount rate downward.

These two methods can, of course, yield exactly the same resulting value for the liability. To see this, assume that the amount of capital attributable to claims risks is 10% of the liability at any point in time, and the cost of capital rate is 8% of the amount of capital. The cash flow adjustment for any period would be 10% x 8% = 0.8% of the value of the liability at the beginning of the period. Exactly the same valuation is arrived at without making any adjustment to cash flows if one instead subtracts 0.8% from the discount rate used when determining the present value of expected cash flows[12]. We will call this discount rate adjustment s_{clm}. The discount rate to be used would then be $i_f - s_{clm}$.

While the above methods are simple enough mathematically, there can be some difficulty in estimating both the portion of an insurer's total capital that is attributable solely to claims risks and the cost of capital rate. For example, although total capital may be more observable (and the total cost of capital easier to estimate), most insurers retain some investment risk in addition to the claims risks that they take on, and their total capital includes the amount attributable to investment risks.

Some actuaries point out that an alternative methodology allows calibration of the discount rate adjustment by using two quantities that are sometimes easier to estimate – the insurer's total cost of capital and its expected total investment return (i.e., the portfolio rate[13]). The argument is that if one subtracts the full cost of capital (expressed as a yield spread) from the portfolio rate, the result is an appropriate discount rate for insurance liabilities and no cash flow adjustment is required to include a provision for risk.

This "portfolio rate" methodology can be shown to fall within CON 7 guidelines as long as certain relationships are enforced. Essentially, the method assumes that:

$$i_{portfolio} - s_{capital\,cost} = i_f - s_{clm}$$

[11] The cost of capital is the market price for obtaining capital that is to be put at risk. The providers of capital to an insurer expect an investment return higher than the risk free interest rate because their funds have been put at risk. The excess of the investor's expected investment return over the risk free rate is the estimated cost of capital rate. The cost of capital in dollars (or appropriate currency) is the cost of capital rate times the amount of capital required. This amount can be added to liability cash flows in each future time period as a provision for the market price of risk.

[12] This can be demonstrated by example. Suppose the liability at the beginning of the period is L, and the expected claim payment at the end of the period is C. Suppose the financing rate is 5.0%, and the cost of capital is 0.8% of the liability. The cost of capital can be treated as an addition to cash flow, in which case we have L = (C + .008L) / (1.05), using the financing rate as the discount rate. One can re-arrange this equation to be L = C / (1.05 - .008), in which case the cost of capital appears as a reduction to the discount rate rather than a cash flow adjustment. The liability value is the same no matter which formula is used.

[13] Sometimes the term "portfolio rate" is used to refer to a measure of investment return on an amortized cost bais. The usage here is different; "portfolio rate" refers to the expected total return on market value for the portfolio. This estimate depends on current market conditions and is therefore appropriate for market-consistent valuation if used properly.

It is not at first obvious why this relationship should hold. To understand why, let us enumerate all of the risks that need to be reflected in the company's cost of capital. The risks and the interest rate spreads that reflects their market prices are:

s_{clm} spread for claims risks retained by the insurer (adds to cost of capital)

s_{inv} spread for investment risks taken by the insurer (adds to cost of capital)

s_{credit} spread for credit risk accepted by the policyholder (this is the option to default, and if reflected, it reduces the cost of capital)

$s_{liquidity}$ spread for liquidity risk accepted by the policyholder (reduces the cost of capital)

The total cost of capital is then $s_{capital\cos t} = s_{clm} + s_{inv} - s_{credit} - s_{liquidity}$.

Financial economic theory tells us that the expected yield spread on a risky investment should be equal to the market price for accepting the risk of the investment. Since we assume the market price of investment risk is s_{inv}, that could suggest that $s_{inv} = i_{portfolio} - i_{rf}$

We can now write

$$\begin{aligned}
i_{portfolio} - s_{capital\cos t} &= i_{portfolio} - (s_{clm} + s_{inv} - s_{credit} - s_{liquidity}) \\
&= i_{portfolio} - (s_{clm} + (i_{portfolio} - i_{rf}) - s_{credit} - s_{liquidity}) \\
&= i_{rf} - (s_{clm} - s_{credit} - s_{liquidity}) \\
&= (i_f - (s_{credit} + s_{liquidity})) - (s_{clm} - s_{credit} - s_{liquidity}) \\
&= i_f - s_{clm}
\end{aligned}$$

to demonstrate the equality stated earlier.

Objections to the use of the "portfolio rate method" have often been based on the idea that the value of an insurer's liabilities should not depend on its investment strategy. Since the portfolio rate does depend on investment strategy, its use in any way suggests that the valuation depends on the strategy. However, the portfolio rate method as described here includes adjustment for the full investment risk through the cost of capital adjustment. Any increase in investment risk that would lead to a higher portfolio rate also leads to a higher cost of capital, so that the quantity $i_{portfolio} - s_{capital\cos t}$ does not change, at least in theory. As a practical matter, one can check whether $i_{portfolio} - s_{capital\cos t} < i_f$ to determine whether the portfolio rate method is being applied in an appropriate way[14]. When properly applied, the "portfolio rate method" is conceptually consistent with CON7, and may have the advantage of calibrating to more easily estimated quantities.

[14] An exception to this relationship occurs for insurance contracts that contain non-guaranteed investment elements, as described in the next section.

The cost of capital that is used as the provision for risk in the portfolio rate method is not always converted into an interest rate spread. In some cases it is applied as an addition to projected cash flows. Under this variation, the portfolio rate is used directly as the discount rate, but full provision for risk is made through the adjustment to cash flows.

1.3 Risks retained by the policyholder: non-guaranteed elements in insurance contracts

Many insurance contracts include elements that are not guaranteed. For example, an insurer may guarantee that insurance coverage can always be renewed or extended for another time period, but may not guarantee the premium rate for the renewal period. Alternately, an insurer may offer a plan of insurance that returns a portion of that premium on a non-guaranteed basis at the end of the coverage period if and only if claims experience is favorable. The premium for such a plan would, of course, be higher than that for a plan that offers no such potential non-guaranteed benefit.

One can readily see that such non-guaranteed elements tend to reduce the risk of the contract to the insurer. They do so by shifting some risk to the customer. In the latter example, the customer's risk is the uncertainty about the size of the non-guaranteed payment to be received at the end of the coverage period.

In our discussion concerning the discount rate, a special focus needs to be placed on non-guaranteed investment elements in insurance contracts. Many insurance contracts involve an investment (or deposit) element. The contract may include a deposit or fund amount that the customer owns and on which interest is credited. When the interest credited to the fund is not fully guaranteed but depends in some way on the performance of a portfolio of assets, we have a non-guaranteed investment element inside an insurance contract.

To understand valuation of an insurance contract with a non-guaranteed investment element, it helps to start by thinking about a pure investment contract. A pure investment contract simply passes the results of an investment portfolio directly to the contract owner, so we will refer to it as a pure pass-through. The liability for such a contract is typically the current account balance[15].

In some cases, the investment element inside an insurance contract does operate very much like a pure pass-through. In the US, such contracts are called Variable or Separate Account contracts. In the UK, the term is unit-linked.[16]

The more interesting case is an investment element that is not a pure pass-through. The insurer may provide a minimum guaranteed interest rate that will be credited, and then provide the customer with non-guaranteed additional interest credits if investment performance is good. The additional interest credits may be based directly on current market performance or may be spread over time based on an amortized-cost measure of return. In either case, the insurer will charge a fee for the guarantee of a minimum credited rate. The fee is conceptually related to the cost of

[15] That is, before adjustments for fees, expenses, or other elements of the contract.

[16] These contracts typically include certain fees that are subtracted from the account value each period. So while they are not pure pass-throughs because part of the investment return is held back in the form of fees, they still have the characteristic that fluctuations in investment return (and the corresponding risk) are passed through to the account value.

the capital required to support the guarantee. The lower or weaker the guarantee, the lower the fee, and the closer we get to a pure investment pass-through contract.

With this in mind, let's revisit the "portfolio rate method" as described in the previous section. The discount rate including full adjustment for risk was $i_{portfolio} - s_{capital\cos t}$ and was to be strictly less than the financing rate, so we could check that $i_{portfolio} - s_{capital\cos t} < i_f$. However, when non-guaranteed elements are included in insurance contracts, they provide a means of reducing the insurer's risk and therefore reducing its capital cost without necessarily reducing either the portfolio rate or the financing rate. As a result, we could have a discount rate that exceeds the financing rate so that $i_{portfolio} - s_{capital\cos t} \geq i_f$.

Now, even though the discount rate is based on the portfolio rate and may exceed the financing rate, this does not mean that the resulting liability value depends on the company's investment strategy. There is an offset to the excess of the discount rate over the financing rate. The offset is the additional projected liability cash flows that arise from the non-guaranteed interest credited to the customer. The reduction in capital cost attributable to the non-guaranteed elements is typically passed through to the customer as an increase in expected (but non-guaranteed) benefits. Any change in investment strategy that increases the discount rate under this methodology is offset by an increase in projected cash flows from non-guaranteed benefits, so the liability value is at least theoretically not sensitive to the investment strategy.

As a result, when non-guaranteed elements with some sort of investment return pass-through are present, it can be appropriate to use a discount rate in excess of the financing rate as long as the projected cash flows include the pass-through of the additional investment return net of capital cost.

An alternative method for valuation of such contracts is to assume the portfolio earns the risk-free rate (or the financing rate) and to project the non-guaranteed benefit amounts that would be paid with that level of investment return. Such a method is consistent with the reasoning above and with CON 7, but is less realistic because it requires a projection that alters current non-guaranteed crediting rates away from the actual rates currently being paid. This is important, because the behavior of the owners of contracts with non-guaranteed investment elements depends on the level of non-guaranteed amounts being paid. If these amounts are not competitive, contract owners may terminate their contracts. Since contract-owner behavior is typically reflected in cash flow models used for valuation, a projection that alters current non-guaranteed crediting rates away from the actual rates being paid also alters assumed behavior, thereby distorting the cash flow projection used for valuation unless policy-owner behavior algorithms are adjusted.[17]

[17] This problem is frequently encountered in risk-neutral valuations of spread-managed business where the credited rate is assumed to be the portfolio rate less a spread. Since all assets are assumed to earn the risk-free rate(s), the resulting modeled credited rate will be below the risk free-rate. If policyholder behavior assumptions (such as surrender rates) are not appropriately translated into a risk neutral environment, excess surrenders and early exercise of policyholder options might be triggered even in the base scenario and in those scenarios that vary little from the base. It should be further noted that not all actuaries believe policyholder behavior algorithms should be altered in a risk-neutral valuation; these actuaries believe that any resulting anomalous policyholder activity is part of such a valuation.

Section 2 – Interest rate risk under stochastic valuation methods

The accounting literature discusses valuation techniques that involve probability-weighting of several different future outcomes. In CON 7 this is called the "expected cash flows" method, and in the IASB exposure draft on fair valuation, this is called the "expected present value technique". In both cases, the "expected" cash flows are determined as a probability-weighted average of the outcomes of various scenarios, and the valuation is done by discounting the "expected" cash flows. While CON 7 is silent on how to provide for risk under this method, the IASB exposure draft suggests two methods, one in which the discount rate is adjusted away from the risk-free rate and one in which the risk-free rate is used for discounting but the expected cash flows are adjusted to "certainty equivalents".

These methods are reasonable, but they are not the only methods that are in common use. We wish to explain two variations on these methods that are in common use. The variations are 1) the use of probabilities other than the "real" probabilities, and 2) the use of discount rates that vary by scenario.

2.1 Probabilities other than the "real" probabilities

As noted above, IASB ED 2009/5 mentions two methods that can be used to include a provision for risk under the "expected present value technique". The two methods involve either an adjusted discount rate (other than the risk-free rate) or adjustments to the expected cash flows. A third technique not mentioned in ED 2009/5 is to adjust the probabilities of the scenarios to give adverse outcomes greater probability weight. When this is done, the discount rate can be the risk-free rate because the provision for risk is provided through the probability weighting, which adjusts the cash flows to certainty equivalents.

The adjustment of probabilities, in combination with discounting at the risk-free rate, is the theoretical basis of the widely-used Black-Scholes method for valuation of stock options.

Consider the valuation of an option to buy a stock at a price of $100 per share any time within the next year. Suppose the current market price of the stock is $95 per share. The value of the option is based on the probability of the price rising over $100 per share within the coming year. This probability depends on the "volatility" of the stock price.

Under the Black-Scholes method, one uses the observed market value of options to work backwards to determine the "implied" volatility that is consistent with the observed market price. That implied volatility, calibrated to market prices of options available in the market, can be used in valuation of options for which a market price is not available; say, options with different strike prices or options with different expiry dates.

The important concept here is that the "implied" volatility is not the real volatility, and the probability of the option having value based on the "implied" volatility is not the real probability of the option having value. The "implied" volatility is a biased probability that includes an adjustment to reflect the market price of risk. One can use the "implied" volatility and the associated biased probabilities to determine market-consistent prices that include provision for risk without ever needing to know what the real probabilities are.

It should be noted that while valuations calibrated in this manner include a provision for risk, the exact size of the provision for risk is not known, and the method provides no way to determine it. Therefore, any accounting requirement to disclose the size of the provision for risk in the valuation can only be met via a rough approximation that can be less reliable that the estimated market-consistent price itself.

The Black-Scholes method, and other methods that involve use of adjusted probabilities, can be considered variants of "method 1 of the expected present value technique" as outlined in FAS 157 Appendix B and IASB ED 2009/5 Appendix C. The adjusted probabilities are used to determine risk-adjusted expected cash flows, which are then discounted at the risk-free rate. The important aspect of this family of variants is that the method of calibration to market prices bypasses any need to determine or specify the size of the risk margin that is included.

2.2 Discount rates that vary by scenario

An added variation on the "expected present value" method comes into play in valuation of items whose cash flows depend on the level of future interest rates. Common methods for valuation of such instruments involve not only adjusted probabilities, but also discount rates that depend on the scenario of future interest rates.

A common example of such an item is a fixed-rate home mortgage that can be prepaid without penalty at any time. Such a mortgage is more likely to be prepaid if interest rates are low (allowing the mortgagor to re-finance at a lower rate) than if interest rates are high. Therefore the cash flows from such a mortgage are sensitive to the level of future interest rates. We will use an example based on a prepayable mortgage to illustrate how the combination of adjusted probabilities and scenario-specific discount rates is often used to include a provision for the market price of the prepayment risk. Similar techniques are often applied in valuation of insurance contracts with cash flows that depend on the level of future interest rates. However an example of such a contract would be much more complex. The same principles and methods can be illustrated much more simply in the context of a prepayable mortgage.

For our example, we focus on a simple fixed-rate mortgage that requires annual payment of interest, with a balloon payment to repay the full principal at the end of its term. The mortgage will have a principal amount of $1000, an interest rate of 5.122%, and a maturity date of two years after the valuation date. The contractual cash flows are $51.22 at the end of one year and $1051.22 at the end of two years.

We also assume that the risk-free yield curve on the valuation date is 5.0% for the first year and 5.25% for the second year.

If there were no option to prepay, the present value at the risk-free yield curve of $51.22 due in one year and $1051.22 due in two years is $1000, and this would be the current value of the mortgage.

However, if there is an option to prepay at the end of one year without penalty, one might expect that if market interest rates decline, many mortgage holders would prepay and refinance their mortgage at the lower interest rate. This option to prepay is a risk to an institution that holds the mortgage as an asset. Market provision for this risk should reduce the market value of the prepayable mortgage below $1000.

To value the prepayable mortgage we will, as a simplified example, use the expected present value technique with two scenarios. Under scenario 1 the risk-free rate rises from 5% to 6% after one year. Under scenario 2 the risk-free rate falls from 5% to 4% after one year.

The first step in applying this technique is to calibrate the implied risk-neutral probability weights to market prices. The implied probability weights must be such that they properly price a fixed and certain cash flow at the end of two years, when path-specific discounting of the scenario cash flows is used.

Exhibit 1 shows how this is done. Each line in Exhibit 1 corresponds to a present value calculation along a scenario. Example 1.1 shows a single scenario calculation of the present value of a fixed and certain cash flow of $1000 at the end of two years, with discounting at the current risk-free yield curve. The present value is $904.88.

Example 1.2 shows what happens if we apply path-specific discounting using our two scenarios, and guess at the probabilities. As an initial guess we specify probabilities of 50% for each scenario. The probability-weighted present value is $907.11, which is incorrect. We therefore need to adjust the probabilities to produce the proper probability-weighted value of $904.88.

Example 1.3 shows the corrected probabilities. Note that for the institution that holds a fixed-rate instrument as an asset, an increase in market interest rates is an “adverse” scenario because the return on the asset is locked and does not rise with the market. The probability of the adverse scenario is increased to 62.9454% from 50%, thereby giving more weight to the adverse scenario. When these probability weights are used, the probability-weighted present value is $904.88, as it should be. These adjusted probabilities are often termed the “risk-neutral” probabilities.

This process of calibrating the scenarios and probabilities to market prices is vitally important when including a provision for risk using “expected present value” techniques.
Now that our scenarios and probabilities have been calibrated, we can use them to value the mortgage, first assuming no prepayment risk and then assuming significant prepayment risk.

Examples 2.1 and 2.2 are valuations assuming no prepayment risk. Example 2.1 does not use path-specific discount rates, and Example 2.2 does use path-specific discounting. Since the cash flows are fixed and do not depend on the scenario, the probability-weighted present value is the same under both methods at $1000.00.

Examples 2.3 and 2.4 are valuations assuming significant prepayment risk. We assume that if interest rates fall to 4% at the end of year 1, fully half of the mortgage principal will be prepaid, of a prepayment of $500. In that case the cash flows are $551.22 at the end of year 1 and $525.61 at the end of year 2.

Example 2.3 does not use path-specific discounting, and obtains a probability-weighted present value of $1000.22. Clearly this result must be incorrect because it is greater than the value of the non-prepayable mortgage. The fundamental reason it is incorrect is that it includes no provision for risk. Example 2.4 uses path-specific discounting to obtain a value of $998.10. This clearly does make provision for the prepayment risk. The value of the prepayment option is $1000 - $998.10, or $1.90.

This example is not intended to fully explain the theory behind use of scenario-specific discount rates. However, the reader should take away the following main ideas.

- Scenario-specific discounting is commonly used to include a provision for risk when future cash flows depend on the level of future interest rates.
- Scenario-specific discounting is used only in combination with careful calibration of the scenarios and the implied risk-neutral probability weights to market prices.

The second point above is particularly important because it is critical to the theory that supports scenario-specific discounting. There are many ways to calibrate the scenarios and the probability weights to market prices. A short description of a method commonly used in valuation of insurance liabilities may be helpful.

In valuation of insurance liabilities whose cash flows depend on future interest rates, one frequently uses a very large number of interest rate scenarios for many months or years into the future. Rather than adjusting the probabilities of the scenarios, the calibration process adjusts the path of future interest rates in each scenario. This is done by adding a calibrated "drift" to the change in interest rates each period before any stochastic or random change is applied by the scenario generator. In that way, even though the probabilities of the scenarios are all treated as equal, the number of scenarios with increasing or decreasing interest rates is adjusted, thereby adjusting the overall probability that interest rates will rise or fall. The "drift" is calibrated so that the probability-weighted present value of a fixed and certain cash flow at any future date is equal to its current market price on the valuation date. To accomplish this for all future dates, the "drift" is not a constant, but a series of calibrated amounts for each future time period.

Lastly, an important aspect of scenario-specific discounting is that the scenario-specific discount rate for each period in each scenario is the short-term one-period risk-free rate. While a scenario generator may provide a full yield curve at every point in time along each scenario, it is the path of the short-term one-period rates that needs to be used for discounting.

Summary

The purpose of this paper has been to outline several methods that can be used in valuation of items that involve risk and uncertainty, emphasizing their compliance with the principles of fair valuation as outlined in such existing accounting pronouncements as CON 7. There has been concern among some actuaries that certain methods that we believe are consistent with CON 7 could, in future accounting standards, be disallowed. There has been particular concern over methods for valuation of insurance liabilities that involve scenario path-specific discounting, or use of the insurer's portfolio rate, or methods used when non-guaranteed investment elements are present. These methods have been shown here to be consistent with CON 7 when properly applied.

A short paper such as this cannot possibly cover all the valuation methods that are in use, and neither can an accounting standard. That is why CON 7 is so important – it states the principles that must be followed, yet allows flexibility in the application of those principles as needed and appropriate. It is our hope that future accounting standards continue the practice of explicitly stating that alternate approaches for discounting and fair valuation are allowed.

Exhibit 1 - Valuation of a fixed and certain cash flow: Calibration of Risk-Neutral Probability Weights

Scenario	Cash flows End yr 1	Cash flows End yr 2	Discount rate during Year 1	Discount rate during Year 2	Discount factor for: End yr 1	Discount factor for: End yr 2	Scenario Present Value	Probability Weight	Weighted Present Value
Example 1.1 No path-specific discounting									
1	$ -	$ 1,000	5.00%	5.25%	0.952381	0.904875	$ 904.88	100.00%	**$ 904.88**
Example 1.2 Path-specific discounting, "real" probabilities									
1	$ -	$ 1,000	5.00%	6.00%	0.952381	0.898473	$ 898.47	50.0%	$ 449.24
2	$ -	$ 1,000	5.00%	4.00%	0.952381	0.915751	$ 915.75	50.0%	$ 457.88
								Total weighted present value:	**$ 907.11**
									Incorrect!
Example 1.3 Path-specific discounting, "risk-neutral" probabilities									
1	$ -	$ 1,000	5.00%	6.00%	0.952381	0.898473	$ 898.47	62.9454%	$ 565.55
2	$ -	$ 1,000	5.00%	4.00%	0.952381	0.915751	$ 915.75	37.0546%	$ 339.33
								Total weighted present value:	**$ 904.88**

When discounting using the scenario path of risk free rates, one must use calibrated "risk-neutral" probability weights.

Exhibit 2 - Valuation of $1000 mortgage

Scenario	Cash flows End yr 1	Cash flows End yr 2	Discount rate during Year 1	Discount rate during Year 2	Discount factor for: End yr 1	Discount factor for: End yr 2	Scenario Present Value	Probability Weight	Weighted Present Value
If cash flows have no prepayment risk:									
Example 2.1 No path-specific discounting									
1	$ 51.22	$ 1,051.22	5.00%	5.25%	0.952381	0.904875	$ 1,000.00	62.9454%	$ 629.46
2	$ 51.22	$ 1,051.22	5.00%	5.25%	0.952381	0.904875	$ 1,000.00	37.0546%	$ 370.55
								Total weighted present value:	**$ 1,000.00**
Example 2.2 With path-specific discounting									
1	$ 51.22	$ 1,051.22	5.00%	6.00%	0.952381	0.898473	$ 993.27	62.9454%	$ 625.22
2	$ 51.22	$ 1,051.22	5.00%	4.00%	0.952381	0.915751	$ 1,011.44	37.0546%	$ 374.78
								Total weighted present value:	**$ 1,000.00**
If cash flows have significant prepayment risk:									
Example 2.3 No path-specific discounting									
1	$ 51.22	$ 1,051.22	5.00%	5.25%	0.952381	0.904875	$ 1,000.00	62.9454%	$ 629.46
2	$ 551.22	$ 525.61	5.00%	5.25%	0.952381	0.904875	$ 1,000.58	37.0546%	$ 370.76
								Total weighted present value:	**$ 1,000.22**
									Incorrect! No provision for risk.
Example 2.4 With path-specific discounting									
1	$ 51.22	$ 1,051.22	5.00%	6.00%	0.952381	0.898473	$ 993.27	62.9454%	$ 625.22
2	$ 551.22	$ 525.61	5.00%	4.00%	0.952381	0.915751	$ 1,006.30	37.0546%	$ 372.88
								Total weighted present value:	**$ 998.10**
									Correct! The value of the option to prepay is $1.90.

Bibliography

This bibliography lists some textbooks and a monograph relevant to the subject of this white paper. The material in these books provides an indication of the wide variety of methods that have been developed and are being used in valuations that involve risk and uncertainty.

Interest Rate Modeling, a textbook by Jessica James and Nick Webber. John Wiley & Sons, 2000. Part III of the book is titled “Valuation Methods”.

Derivatives Markets, a textbook by Robert L. McDonald. Addison Wesley, 2006. Part 5 of the book is titled “Advanced Pricing Theory”.

Models for Quantifying Risk, a textbook by Robin Cunningham, Thomas Herzog, and Richard L. London. ACTEX publications, 2005. This book covers a wide variety of models used to characterize and quantify insurance risks.

Insurance Risk Models, a textbook by Harry H. Panjer and Gordon E. Willmot. Society of Actuaries, 1992. This is a classic educational text covering a wide variety of models used to characterize and quantify insurance risks.

Fair Valuation of Insurance Liabilities: Principles and Methods. This 2002 public policy monograph by the American Academy of Actuaries provides an introductory-level survey of many of the methods described in textbooks and other literature.

Measurement of Liabilities for Insurance Contracts: Current Estimates and Risk Margins. This 2009 International Actuarial Research Paper was prepared by the ad hoc Risk Margin Work Group of the International Actuarial Association.